NOTES AND CORRESPONDENCE

Precipitation Modulation by Large-Scale Inertia-Gravity Waves

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Precipitation Modulation by Large-Scale Inertia-Gravity Waves

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Abstract

Observations and a simplified theoretical model suggest that large-scale inertia-gravity waves can interact significantly with large-scale precipitation in the atmosphere.

1. Introduction

In the atmosphere, inertia-gravity waves tend to have small kinetic energy compared to the balanced, quasi-geostrophic flow component. However, in contrast to the quasi-geostrophic flow component, these waves do not tend to be quasi-nondivergent in horizontal planes. Thus, besides being important in geostrophic adjustment (Winninghoff, 1968; and Blumen, 1972), these waves might contribute significantly to *large-scale* divergence and associated *vertical motions*. By "large-scale", we mean flow components with horizontal scales at least 0(1,000 km). It follows that they might interact significantly with large-scale precipitation.

In Section 2, we discuss observations that, indeed, apparently reflect large-scale precipitation modulation by large-scale inertia-gravity waves. This modulation implies oscillating latent heat release which, as shown in Section 3, can drive or modify large-scale inertia-gravity waves with vertical motions sufficient to modulate large-scale precipitation.

2. Observations

Due to the lack of time resolution and accuracy, large-scale inertia-gravity waves would tend to appear as noise in much of the available atmospheric data. Also, the closeness of the diurnal and/or semi-diurnal periods to the natural inertia-gravity period can lead to ambiguity in the interpretation of observations. Further, relatively large background geostrophic flow can obscure the inertia-gravity wave motion even when its oscillating divergence is a significant component of the total large-scale divergence field. However, precipitation is an important weather parameter that is strongly influenced by the divergence field. Therefore, high frequency, tipping-bucket rainfall data should reflect the significance of the large-scale inertia-gravity wave contribution to the large-scale divergence field.

Observations from the Project Experiment of 1969 have sufficient time resolution to reveal large-scale inertia-gravity waves. Many interesting features in these observations have been reported. Matsumoto, *et al.* (1970) note the presence of a significant ageostrophic jet that fluctuates rapidly in time and is closely related to precipitation. Yoshizumi (1977) notes the occurrence of disturbances whose divergence and vorticity are of comparable magnitude. In many of these observations, there is a dominant time scale of about 20 hours. Perhaps the most striking observations are the time sequences of precipitation from disturbances along a subtropical front during July 9 through July 12, 1969, as described by Matsumoto, *et al.* (1970) and by Akiyama (1978). Of particular interest here is the time history of the areal mean rainfall (see Figure 1) and accompanying radar cloud pictures (Akiyama, 1978; see Figure 6). A dominant feature of this time sequence is an

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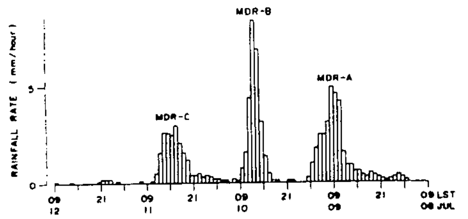

Fig. 1 Time history of a real mean hourly rainfall rate averaged on all guage data (65 stations) from a wet disturbance. Reproduced from Fig. 4 of Akiyama (1978).

oscillation with a period near 20 hours, which remains strong through at least three peaks. The areal mean rainfall varies from less than 1 mm per hour, between the peaks, to about 5 mm per hour. This implies rapid fluctuations of large-scale latent heat release, sufficient to drive or significantly modify large-scale inertia-gravity waves with strong vertical motion, according to the analysis in Section 3. From the radar cloud pictures, the region of cloud cover also appears to expand and contract significantly with period about 20 hours. This 20-hour period is close to the natural large-scale inertia-gravity period in the region. (Although the inertia period is about 22 hours in the region, about 33° latitude, the buoyancy effects would generally reduce this period to below 22 hours in regions of positive static stability, depending on the horizontal scale of the oscillation and the magnitude of the areal mean static stability. Note that the effective static stability should be rather weak in the actively convecting region of interest.)

More recently, Brunet (1974) found evidence of such oscillations in the areal mean rainfall from Hurricane Agnes, although they were apparently less important than in the Project Experiment of 1969 observations. Jakobsson (1973) analyzed much longer time sequences of pressure at a stationary ship in the North Atlantic Ocean. During one winter period, he found a significant peak in the inertia-gravity frequency range. It is expected that analysis of precipitation data (if available) from disturbances of that period would reveal an even more significant peak in the inertia-gravity range.

3. A simplified model to estimate vertical velocity in large-scale heating-induced inertia-gravity waves

The observations in Section 2 suggest that large-scale precipitation can be significantly modulated by the vertical motions associated with large-scale inertia-gravity waves. From the time-dependent precipitation, we can estimate an associated time-dependent, large-scale condensation heating. Large-scale inertia-gravity waves are generated by such time-dependent, large-scale heating. A basic question is whether the vertical motions associated with such heating-induced inertia-gravity waves are sufficient to significantly modulate the precipitation. We, therefore, use a simplified model to estimate these vertical motions.

The following simplified approach does not consider the effects of buoyancy on the flow. In a stably stratified region, these effects would act to disperse the energy and reduce the period of the oscillations. However, inertia-gravity waves are likely to be most significant when there is unusually rapid increase of heating rate of a propagating air mass, such as expected in East Coast winter storms. In these cases, the static stability is usually relatively weak in the heated region, so the buoyancy effects should be secondary. They should not affect the order-of-magnitude of the inertia-gravity wave amplitudes, which is the main interest here. (To include the buoyancy effects, a much more complex multi-dimensional model would be necessary, with boundary layer phenomena included.)

We start with the familiar vorticity-divergence formulation of the horizontal equations of motion,

$$\frac{d\zeta}{dt}=-(f_0+\zeta)D$$

$$\frac{dD}{dt}=D^2-2J(u,v)+f_0\zeta-\nabla_p{}^2\phi\,, \qquad (1)$$

where f_0 = Coriolis parameter (assumed constant); ζ = relative vorticity; D = divergence; ϕ = geopotential; and J is the Jacobian operator. Scaling indicates that the nonlinear term, $D^2-2J(u,v)$, is small compared to dD/dt. (The time scale associated with the neglected term is much larger than inertia time scale, and we are interested in the inertia-gravity component of motion.) The Jacobian term drops out identically if one ignores variations along a frontal line. Ignoring the nonlinear terms, the following equation is derived from Eqs. (1).

$$\frac{d^2D}{dt^2}+f_0(f_0+\zeta)D=-\frac{d}{dt}(\nabla_p{}^2\phi) \qquad (2)$$

If one considers flow near a disturbance center, the Lagrangian frame assumed by Eq. (2) should closely follow the disturbance. The right-hand side of Eq. (2) is affected by the rate of *localized* heating following the motion. Below, we will use Eq. (2), together with estimates of heating rates, to estimate the order-of-magnitude of heating-induced vertical motions.

It should be noted that in rapidly propagating disturbances, a free inertia-gravity oscillation would not appear as a distinct peak near the natural frequency if the time series is from a stationary frame. Instead, two peaks would tend to appear: one above the natural inertia-gravity frequency and one below it. A more appropriate frame of reference for studying inertia-gravity waves appears to be one following a disturbance, since this would tend to approximate a mean Lagrangian frame, following the mean fluid elemental motions averaged over the disturbance region. (However, for a quasi-stationary disturbance, a stationary frame is an acceptable approximation.)

We now consider the effect of heating on the large-scale divergence "D" in Eq. (2). There are several responses the atmosphere will make in general to localized heating. First, the atmosphere will tend to rise, and an associated "adiabatic cooling" will occur as the air mass moves upward to lower pressure. In a statically stable environment, this will tend to cool the air at any given pressure level, thereby partly compensating the heating. Second, the non-compensated heating will raise the temperature and lower the density at each pressure level in the heated air mass, thereby reducing the hydrostatic thickness between any two pressure levels. Third, if the heating leads to convective instability, rapid overturnings will occur, resulting in a re-distribution of mass in the overturned region. Note that this overturning would not significantly change the mean temperature of the overturned region unless latent heat were released by the overturnings; the overturnings by themselves only transport heat, so the upper part of the overturned region is warmed and the lower part cooled.

Now, we consider two extreme cases. If the static stability is large in the sense that large-scale gravity waves (same scale as the heating) have much higher frequency than inertia oscillations, the "adiabatic cooling compensation" will be almost complete, at least after averaging out high-frequency gravity wave oscillations. Almost complete release of the available potential energy generated by the localized heating is obtained in this case. If the large-scale heating has time scale comparable or shorter than the large-scale gravity waves, the oscillations will be significant until their energy is dispersed. In such a case, the resulting oscillations can contribute significantly to the large-scale divergence field. (For example, the heating rate due to the condensation of 1 cm of precipitation in a layer between 750 mb and 1,000 mb would be sufficient to raise the layer mean temperature by 10°C. To compensate this by "adiabatic cooling" normally requires a vertical displacement of about 3 km. If the heating and compensation occurred in 5 hours, the mean vertical velocity would be about 15 cm/sec.)

In the other extreme case, the static stability is small in the sense that large-scale gravity waves have lower frequency than inertia oscillations. In this case, the rotational constraint is strong, and there is relatively little "adiabatic cooling compensation" during an inertia oscillation period. In cases of strong heating from below, such as expected during East Coast winter storms, the atmosphere is probably closer to this second extreme than the first. In such cases, we can neglect such compensation and use Eq. (2) to estimate D and the associated vertical velocity, as follows.

First, using the hydrostatic thickness relation

$$z(p_2, x, y) - z(p_1, x, y) = -\frac{R}{p_0^{\kappa} g}\int_{p_1}^{p_2} \cdot\, \theta(p, x, y)\, d\left(\frac{p^{\kappa}}{\kappa}\right), \qquad (3)$$

and the heat capacity of air and the heat of condensation of water vapor, we can show that

$$\Delta z \doteq 100\frac{\mathrm{m}}{\mathrm{cm}} \cdot r, \qquad (4)$$

where Δz = the thickness change in meters and r = water equivalent precipitation in centimeters. Latent heat of fusion would add another 15 m/cm, and sensible heat could add a similar amount. (It is observed that both sensible and latent heat are added to cold air advected over water.) Using Eq. (4), the *r.h.s.* of Eq. (2) may be written

$$\frac{d}{dt}(\nabla_p^2\phi) = O\left(g\nabla_p^2\frac{d}{dt}(\Delta z)\right)$$
$$\doteq g\nabla_p^2\left(100\frac{\mathrm{m}}{\mathrm{cm}}\right)\frac{dr}{dt}. \qquad (5)$$

Thus, we may write

$$\frac{d^2D}{dt^2} + f_0^2 D = O\left\{g\nabla_p^2\left(100\frac{\mathrm{m}}{\mathrm{cm}}\right)\frac{dr}{dt}\right\}$$
$$= O\left(10^4 4\pi^2\frac{g}{L^2}\frac{dr}{dt}\right). \qquad (6)$$

If one assumes $L = O(2{,}000\ \mathrm{km})$, and $w =$

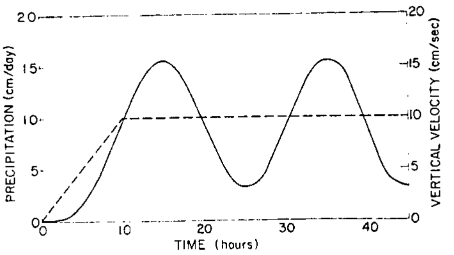

Fig. 2 Dashed curve is idealized time variation of large-scale precipitation during the development of an east-coast winter storm. Solid curve is large-scale vertical velocity response to associated large-scale latent heat release, calculated from a simplified theoretical model.

$O(10^5\ \mathrm{cm}\ D)$, Eq. (6) may be written

$$\frac{d^2w}{dt^2}+f_0^2w=10^{-3}\frac{dr}{dt}\ \mathrm{sec}^{-2} \qquad (7)$$

In Fig. 2, the solid curve is the solution to Eq. (7) assuming balanced initial conditions ($w=dw/dt=0$ at $t=0$) and latent heat release associated with a specified precipitation rate (dashed curve). (The solid curve is reduced by a factor of $\pi/2$ to get a representative large-scale average w.) The assumed condensation rate is qualitatively similar to that expected in East Coast winter storms. Note that there is a significant oscillation of $O(\pm 6\ \mathrm{cm/sec})$ around a long term mean of $O(9\ \mathrm{cm/sec})$. More generally, it is easily demonstrated that when dr/dt changes rapidly (with time scale of $O(2\pi/f_0)$ or less), the solution to Eq. (7) has a significant oscillating component with period $2\pi/f_0$.

Thus, the large-scale heating rates inferred from the precipitation data from the Project Experiment of 1969 (see Figure 1) should generate or modify significant large-scale vertical velocity (associated with large-scale inertia-gravity waves) capable of strongly modulating the large-scale precipitations.

In the above simplified approach, the feedback of large-scale divergence into the heating function has been ignored by specifying (independently) a particular heating function. This feedback is negatively correlated with the low-level D. To include this effect (at least qualitatively) for a given disturbance, one could write

$$\frac{d}{dt}(\Gamma_p{}^2\phi)=q_0-\alpha D, \qquad (8)$$

where $\alpha>0$ and q_0 contains factors other than D affecting the geopotential tendency. If $\alpha>f_0$ $(f_0+\zeta)$, the solution to Eq. (2) would be exponential rather than oscillating. Physically, this would mean that the vorticity increase associated with low-level convergence is not sufficient to balance the increase of geostrophic (or gradient) vorticity associated with the related heating. The result is an "isallobaric divergence" which, along with vorticity, increases with time. The growth of such a disturbance could not be predicted by a balanced model. (Note that such growth is most likely in low latitudes, where f_0 is small and α is relatively large.) Eventually, however, the increase of relative vorticity, which has the same qualitative effects as increasing f_0 (see Eq. (2)), would stabilize the flow. Such stabilization, which should first appear near a disturbance center, may contribute to the formation of the observed, relatively calm regimes near the center of intense tropical storms.

4. Summary and concluding remarks

In the observations discused in Section 2, there is apparently significant large-scale precipitation modulation by large-scale inertia-gravity waves. In considering the possible atmospheric response to the associated large-scale heating modulation, we found (using a simplified theoretical model) that reasonable, time-varying, large-scale heating can lead to significant oscillating vertical motions associated with large-scale inertia-gravity waves. When the heating time scale is comparable to the natural inertia-gravity wave time scale, the oscillations should be most significant. If the heating rate changes relatively slowly, the amplitude of the oscillations is greatly reduced. It, therefore, appears that in rapidly growing disturbances with significant heating, such as East Coast winter storms, significant interaction probably often occurs between large-scale precipitation and large-scale inertia-gravity waves. Although much of the available weather data does not accurately resolve these relatively short-period oscillations, it may be possible to use high-frequency, tipping-bucket rainfall data to derive quantitative values for these waves when they are important. To obtain more detailed understanding of these waves requires detailed observations, together with complicated numerical models (including effects of three-dimensional structure, static stability, and boundary layer phenomena), integrated on large computers.

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References

Akiyama, T., 1978: "Mesoscale Pulsation of Convective Rain in Medium-scale Disturbances Developed in Rain Front". *J. Meteor. Soc. Japan*, 56, 267–283.

Blumen, W., 1972: "Geostrophic Adjustment". *Rev. Geoph. and Space Phy.*, **10**, 485–528.

Brunet, N., 1974: "Time Spectral Analysis of Space-averaged Precipitation", (Master's Thesis, McGill University), 78 pp.

Jakobsson, T., 1973: "Spectral Analysis of Marine Atmospheric Time Series", (Ph.D. Dissertation, McGill University).

Matsumoto, S., and M. Takeuchi, 1970: "On the Structure of the 'Baiu Front' and Associated Intermediate-scale Disturbances in the Lower Atmosphere". *J. Meteor. Soc. Japan*, **48**, 479–491.

Winninghoff, F. J., 1968: "On the Adjustment Toward a Geostrophic Balance in a Simple Primitive Equation Model with Application to the Problems of Initialization and Objective Analysis", (Ph.D. Dissertation, University of California/Los Angeles), 161 pp.

Yoshizumi, S., 1977: "On the Structure of Intermediate-scale Disturbances on the Baiu Front". *J. Meteor. Soc. Japan*, **55**, 107–120.

大規模慣性重力波による降水のモデュレーション

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大気中において大規模慣性重力波と大規模降水との間にかなりの相互作用が起りうる可能性が，観測および簡単化した理論モデルによって示唆される。

NOTES AND CORRESPONDENCE

On the Comparison of Computed Cloud Mass Fluxes with Observations over the GATE Area

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Abstract

The cumulus mass flux obtained by the diagnostic analysis is verified by using digital SMS-1 IR brightness data and precipitation calculated from radar measurements. The time variations of the cloud mass flux due to deep clouds correlate quite well with those of IR brightness and precipitation. The precipitation calculated from the model is comparable to that observed. Relationships of the cloud mass flux, IR brightness and precipitation to the African wave disturbance are also discussed.

1. Introduction

In recent years computations of the cloud mass fluxes based on large-scale heat and moisture budgets and parameterized cloud models have been performed by many authors to study the interaction between cumulus convection and its environment (Yanai *et al.*, 1973; Ogura and Cho, 1973; Nitta, 1975; Johnson, 1976; and others). Using observations from the GARP Atlantic Tropical Experiment (GATE) Nitta (1977, 1978) and Johnson (1978) made similar computations including both convective updrafts and downdrafts.

While these diagnostic studies clarified the important role of the cumulus cloud in the large-scale field, the results of the cloud mass flux computations are dependent on the adopted cloud models and should be verified by other independent observations for clouds. However comparison between computed cloud mass fluxes and other directly observed cloud parameters has not been thoroughly made mainly due to the lack of direct observations for cumulus clouds.

The GATE observation systems have been planned to provide the data for estimating the effects of the smaller-scale systems on the large-scale fields in the tropics and a variety of observations have been carried out to measure different scales of meteorological phenomena. Therefore it is possible to test the computed cloud mass fluxes by different types of observation data obtained in GATE.

This study is an extension of the previous study by Nitta (1978) in which the cloud mass fluxes were computed using the GATE A/B-scale upper-air observations for Phase III. The purpose of this study is to compare the computed cloud mass fluxes with the observed convective activity based on the satellite and radar observations and to verify the cloud mass flux computations.

2. Data

The results of the cloud mass fluxes obtained by Nitta (1978) are used in this study. The cloud mass flux computations have been carried out for each 6 hourly observation time using upper-air observations over the GATE A/B-scale area during Phase III (31 August-18 September 1974). Fig. 1 shows observation networks for A/B- and B-scale areas in GATE. The reader is referred to Nitta (1978) for further details of the computation method.

Digital SMS-1 *IR* brightness data and precipitation calculated from radar measurements are used for comparison with the cloud mass flux. 3-hourly *IR* brightness data averaged over the A/B-scale area were kindly supplied by Dr. M. Murakami of Meteorological Research Institute of Japan. Additional details of the data processing are described by Murakami (1979). Short period variations less than 1 day are filtered out