NOTES AND CORRESPONDENCE

Comments on "A Direct Solution of Poisson's Equation by Generalized Sweep-Out Method"

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Hirota *et al.* (1970) have introduced to meteorological literature a conceptually simple, fast, direct method for solving two-dimensional discrete boundary value problems. (After preprocessing, the method requires $0(10\ n^2)$ operations for an $n \times n$ grid, roughly equivalent to two conventional relaxation sweeps.) Roache (1971) discovered the method independently and discussed its applications and limitations. In a note of response to the Hirota *et al.* paper, McAveney and Leslie (1972) observed that the "generalized sweep-out method" (GSM), as described by Hirota *et al.*, is unsuitable for large grids, due to numerical precision requirements. In the present note it is observed that the GSM can be efficiently applied to large problems by iterating with subdomains.

The method has been applied to solving a Poisson equation on a 56×56 grid. With nine 20×20 overlapping (see below) subdomains, the rate of convergence is about 8 times faster than the corresponding ADI scheme (using a relaxation coefficient of unity in both cases in order to allow proper comparison). Of course, there are faster ADI methods (such as overrelaxation.) However, the ADI method is a special case of this method, and similar improvements should apply to the general method. As applied, residuals decrease 11% per iteration*.

The iterative GSM simply applies the GSM (as described by Hirota, *et al.*) to successive sub-grids or subdomains of the problem to be solved (if the problem is too large for direct solution using the available computer precision). To take full advantage of the available computer precision, one would use subdomains as large as allowed by the available computer precision. Just as with ADI and ordinary relaxation methods, the boundary values of successive subdomains are held fixed, while adjusting the interior point or points such that the finite difference equation is satisfied (or almost satisfied) at the interior points. Thus, the "boundary conditions" of the subdomains are either results from previous iterations, or a guess field.

The most straightforward application uses rectangular subdomains on a rectangular grid. The subdomains must overlap by two lines. In the ADI case, the rectangles reduce to three adjacent lines: an interior line sandwiched by two boundary lines. The interior line is adjusted, with the boundary lines held fixed (thereby serving as "boundary conditions"). The adjusted interior values serve as a boundary line for the next subdomain; thus, there is a two-line overlap. Obviously, the subdomains need not be restricted to three lines (as in ADI methods). Further, the GSM offers an efficient method for solving multiple interior line subdomains. Several advantages of multiple interior line subdomains are evident: the solution of a given subdomain does not destroy the balance at all interior points of the previously solved subdomain (the balances at only one interior line are destroyed); and the total grid coupling is faster.

A key property of the multi-line procedure is that the accuracy of the solution after one iteration (over all subdomains) is completely determined by the accuracy of the "boundary" values applied to the successive subdomains. Thus, unlike ADI and ordinary relaxation methods, the relevant guesses are a small subset of the total number of grid points. Thus, it becomes highly advantageous to extrapolate these values from previous calculations, when possible. Since the number of guess values is relatively small (compared to ADI

* The calculation was performed with double precision arithmetic. Single precision is adequate for 20×20 subdomains, but the convergence rate is slower unless several iterations in each subdomain are performed to relax the roundoff error. Even with 324 5×5 subdomains, the convergence rate is faster than for ADI.

Table 1. Convergence rates of iterated GSM for various subdomain arrangements. Poisson problem with homogeneous Dirichlet boundary conditions is solved on a 56×56 grid, using the 5-point Laplacian. The forcing function is (x^2+y^2-2). N=number of overlapping subdomains; I=number of points in the short direction of the subdomains; J=number of points in the long direction; and R=percent decrease of mean absolute error per double (alternated direction) sweep. Cases using double precision arithmetic (on a CDC 7600 computer) are indicated by an asterisk.

N	I	J	R
54 (ADI)*	3	56	1.3
18*	5	56	3.5
18*	8	29	7.2
18	11	20	8.4
324	5	5	2.0
81	8	8	3.9
36	11	11	5.8
9*	20	20	10.9
4*	29	29	14.5

methods), higher order extrapolations should prove beneficial. Both storage and computation requirements for the guess values are relatively small. This may be the best advantage in application of the iterated GSM method to related sequences of problems for which accurate extrapolation is possible(such as time marching field problems).

Finally, the writer has found that, in solving the Poisson problem in a square region, the optimum procedure is to use the largest possible square subdomains rather than using rectangular subdomains. This maximizes the convergence rate obtainable with a given computer precision. Convergence rates for various subdomains are given in Table 1.

References

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「一般化された掃き出し法によるポアソン方程式の解法について」のコメント

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