

Nonlinear Internal Wave Run-Up on Impermeable Steep Slopes

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Laboratory experiments were conducted to investigate the run-up of internal solitary waves (ISWs) on steep uniform slopes in a two-layered fluid system with a free surface. A 12 m long wave flume, which incorporated a movable vertical gate for generating ISWs, was used in the experiments. A steep uniform slope was modeled at one end of the flume. In the present study we looked at internal waves with small and large amplitudes using a steep uniform slope (from one of $\theta=30, 50, 60, 90, 120, \text{ and } 130$ deg) much longer than those previously published in the literature. Results collected from a wide range of experimental runs show that the run-up height depends on the planar slope, while the breaking depth is only antecedent to the amplitude of an ISW. The overall quantitative agreement with the linear feature aspects of the incident wave amplitude is encouraging. [DOI: 10.1115/1.3168528]

Keywords: internal solitary wave, wave flume, two-layered fluid system, steep uniform slope, run-up height, breaking depth

1 Introduction

There have already been many reports published on the observation of internal solitary waves (ISWs) in the oceans, in lakes, in fjords, and even in the atmosphere. Exploring the properties of the ocean is not always an easy task. ISWs have been observed since the beginning of the 20th century. In fact, some internal waves are large enough in amplitude to cause observable consequences at the surface, suggesting that observation of the ocean surface may help to detect the movement of internal waves. Of course, obser-

vations throughout the water column (at several depths) are required to determine all the characteristic of an internal wave train. For example, maximum wave run-up and possible wave breaking on planar slopes are important to provide an effective transport mechanism that needs to be taken into consideration for several types of engineering structures. In brief, ISWs create significant momentum on continental shelves, which could cause the shoreward transport of planktonic larvae [1], as well as affecting the mixing between subsurface water and bottom water in shallow water [2]. ISW beaching processes, such as shoaling and breaking, also play a very important role in the erosion, suspension, transport, and deposition of sediments [3].

The propagation of ISWs over one-dimensional topography has already been the subject of several investigations. The splitting up of a single solitary wave in deep water by means of a variable slope into a number of solitarylike waves at a shelf was investigated by Kao et al. [4], who carried out experiments on the shoaling process of an ISW over a uniform sloping bottom. They observed that waves of sufficient height were able to break on a slope with a Richardson number of less than 0.25. The slopes in their study were mild, with ratios of 1:16 and 1:9 (vertical: horizontal). Helfrich [5] reported on the conversion of a single solitary wave into boluses, and found that $15 \pm 5\%$ of the energy lost inshore of the breaking point went into vertical mixing. The kinematics of the run-up to the breaking point and the bolus were described as having slope ratios of 1:15, 1:20, and 1:30. Michallet and Ivey [6] measured large-amplitude long waves that occurred at the interface of two miscible fluids (fresh and brine water). They examined the mixing process and quantified the energy loss associated with the breaking of the ISW over a uniformly sloping bottom. For two slightly steeper slope ratios of 1:6 ($\theta=9$ deg) and 1:4.7 ($\theta=12$ deg), they examined the amount of wave energy lost. The maximum estimated loss was approximately 25% when the characteristic length ratio $L_w/L_s=0.5$, where L_s is equal to H/s , and L_w is the characteristic length of the ISW.

Helfrich [5] concluded the function for the run-up to be approximately linear to the amplitude of the incident wave and to be independent of the slope of the seabed. A similar linear relationship between run-up distance and the incident wave height was also reported by Wallace and Wilkinson [7]. However, the experimental results of Saffarinia and Kao [8] differed from those of Helfrich [5] and Wallace and Wilkinson [7]. They found the total wave run-up height to be a linear function of the parameter θ , which was dependent on the seabed slope, the incident wave amplitude, and the total water depth. In all three cases, the run-up distance was defined as from the point of interface-slope intersection to the maximum up-slope reach of the wave. Saffarinia and Kao [8] found the run-up distance to be higher for gentler slopes. Eriksen [9] believed that this higher onslope flow for gentler slopes could be a result of the following two factors: (a) the critical reflection frequency for gentler slopes is closer to the inertial frequency, thus, internal wave energy levels are higher; (b) since the flow near the bottom must be parallel to the slope, the onslope flow must be stronger for a gentler slope.

Most of the abovementioned reports suggest that wave run-up distance depends on either the inclination of the slope or the incident wave amplitude. Except for the cases presented in Ref. [6], almost all the slopes in previous reports and laboratory experiments were less than 10 deg. The ranges of gentle slopes used in prior sources are collected and summarized in Table 1. On the other hand, studies on ISW run-up distance for a wider range of steep uniform slopes and reversed angles are not available. In this paper, we present the results of the run-up of elevation and depression ISWs on uniform slopes exceeding 30 deg and with reversed configurations. The layout of the rest of the paper is as follows: the problem statement and complexity of ISW run-up distance are highlighted in Sec. 1; in Sec. 2 the laboratory experiment is described. In Sec. 3 we define the characteristics and measurement of the ISW, which include run-up height and break-

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Table 1 Slope ranges used in the experiments regarding internal wave reflection from uniform slopes from uniform slopes

Authors	Year	Flume size ($W \times H \times L$) (m)	Waveform	Slopes (deg)	Wave maker	Nonlinearity (a/H)
Kao et al. [4]	1985	$0.36 \times 0.61 \times 9.14$	Depression	4, 6	Vertical gate	0.01–0.2
Wallace and Wilkinson [7]	1988	$0.6 \times 0.75 \times 18$	Periodic	1.7, 3.1	Horizontal paddle	-
Helfrich [5]	1992	$0.4 \times 0.65 \times 18$	Depression	2, 3, 4	Flap-type	0.02–0.09
Saffarinia and Kao [8]	1996	At numerical experiment	Depression	4, 7.9, 15.6	At numerical experiment	<0.22
Michallet and Ivey [6]	1999	$0.25 \times 0.2 \times 2.3$	Depression	4, 9.6, 12	Gate-type	0.1–0.2
Vlasenko and Hutter [19]	2002	At numerical experiment	Depression	0.52–21.8	At numerical experiment	Variables
Umeyama and Shintani [18]	2006	$0.15 \times 0.35 \times 0.65$	Periodic	2	Horizontal paddle	<0.06
Chen et al. [17]	2007	$0.5 \times 0.7 \times 12$	Elevation and depression	30–130	Gate-type	<0.25

ing depth. The results of wave measurement are presented in Sec. 4. Finally we end with a conclusion and discussion.

2 Experimental Set-Up

Laboratory experiments were conducted in a 12 m long steel-framed wave flume, with a rectangular cross section, 0.7 m high by 0.5 m wide, on a polycarbonate bottom panel. A 40 cm two-layer fluid system was prepared in the flume consisting of an upper layer of fresh water above a layer of brine (Fig. 1). The upper layer of fresh water had a density ρ_1 and a depth H_1 . The fresh water body was allowed to stand overnight to allow the temperature to equilibrate with the ambient conditions. The medium of the lower layer was brine with a density ρ_2 , which was premixed separately in a constant head tank. The flume was filled slowly to a depth of H_2 by gravity through several small openings along the bottom of the flume. The brine layer had to be carefully infused beneath the fresh water, with a thin piece of sponge placed at every entry point to ensure uniform diffusion into the two-layer fluid system with minimum disturbance and mixing at the interface. The filling time depended on the thickness of the upper and lower layers. For example, with a discharge of about $600 \text{ cm}^3/\text{s}$ for the fresh water and $160 \text{ cm}^3/\text{s}$ for the brine, it took about 2 h to produce a two-layered system of $H_1/H_2=30/10$ (units in centimeters). A coloring agent was later injected through a set of injectors at the interface of the fluid system, thus, producing a sharp visible interface between the fluids, with the brine layer being beneath the fresh water. The density of the upper water layer was maintained at 999 kg/m^3 and the lower layer was maintained at about 1030 kg/m^3 , producing a stratified fluid environment with a Boussinesq parameter of $\sigma \cong 0.03$.

A removable sluice gate panel for generating internal waves was mounted on the right hand side of the flume. No wave absorbing facilities were introduced at either end of the flume; thus, incident waves were free to reflect back to the test section between the slope and the gate. Devices with a rigid slope and impermeable bottom plane allowed an internal wave to reflect back and forth while propagating in the flume, hence, wave at-

tenuation by energy damping primarily originated from bottom friction exceeded several tens of minutes. ISWs were generated by overturning the interfacial water behind the movable gate. A minipump was used to move a small quantity of fresh water from the main section of the flume to the small compartment behind the vertical gate. A corresponding mass of the brine then slowly moved to the other side of the gate through the narrow gap between the bottom of the gate and the bottom of the flume to maintain a hydrostatic balance. This created a prescribed step difference η_0 in the interface levels on either side of the gate. The initial ISW was generated by raising the movable gate using a pneumatic device controlled by a switch. The ISW overturned the brine, causing the lead solitary wave to propagate in the flume. A primary internal wave developed into a leading solitary wave propagating ahead of a transient dispersive wave train. A rigid steel panel at the other end of the flume served as the uniform slope, the angle of which varied between 30 deg and 130 deg (from one of $\theta=30, 50, 60, 90, 120,$ and 130 deg). The toe of each slope tested was located approximately 8.18–8.95 m from the removable gate. The initial interface was able to support 5–6 test runs, from small to large amplitudes, before the flume had to be drained and refilled.

3 Characteristics of ISW Measurement

In Secs. 3.1 and 3.2 we discuss the images of internal waves from observations at different levels.

3.1 Wave Run-Up: R_w . As an ISW propagates shoreward, the vertical velocity at its crest increases due to an increase in the pressure gradient [10]. For this reason, an internal wave is able to run up a slope after shoaling. The most extensive work on the run-up of an ISW along a sloping seabed has been done with two-layer systems, as reported by Helfrich [5] and Wallace and Wilkinson [7]. The length from the interface-slope intersection to the point where breaking is initiated is defined as the wave run-down stage. Wave run-down is also defined as the lowest vertical height reached by the backwash of a wave before the up-rush of

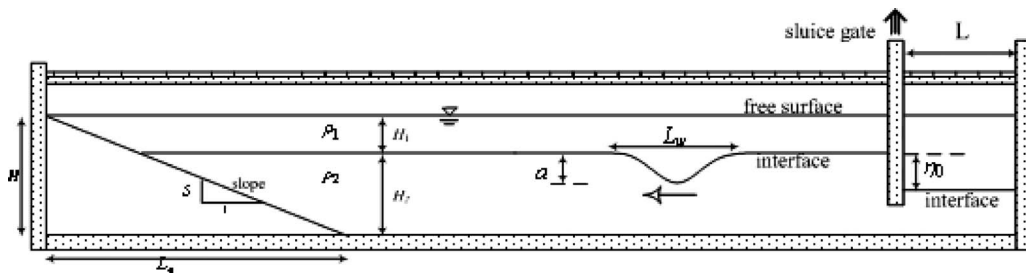


Fig. 1 Schematic view showing the laboratory set-up for internal wave propagation in a two-layer fluid system

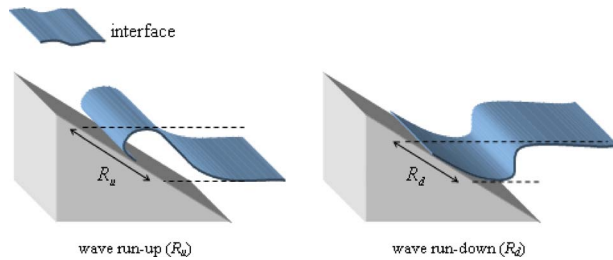


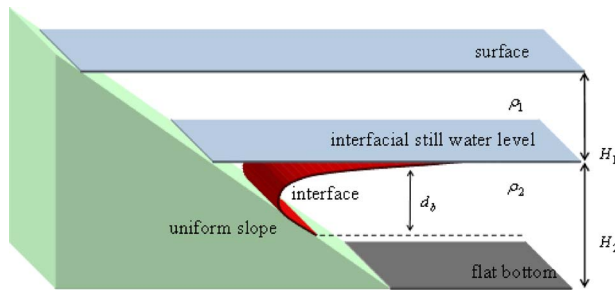
Fig. 2 Schematic drawing of wave run-up and wave run-down phases resulting from wave-slope interaction

the horizontal interface commences its approach to the slope face, and its distance is thus defined as the maximum reach of mixed fluid from the original interface-slope intersection point. These two phases of movement are represented schematically in Fig. 2. The run-up and run-down distances were measured both in the present study.

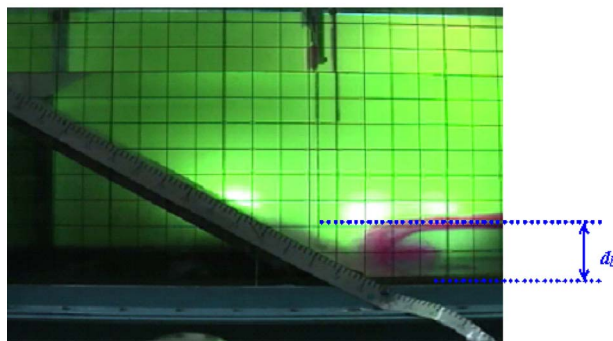
3.2 Breaking Depth: d_b . The breaking depth is a vertical measure related to the distance from the interfacial water level to the interface-slope intersection where the first sign of breaking commences, as illustrated in Fig. 3. A bed that gradually becomes shallower will have a broad surf zone, where large waves break further offshore in the deeper water. A steep bed will have a narrower breaker zone because all the waves tend to reach their breaking depth at around the same place. As a result, breaking depth depends mainly on wave steepness and bottom slope [11].

4 Experimental Results

The run-up of an ISW on a sloping surface is found to be strongly dependent on the slope inclination and wave amplitude (Fig. 4). Generally speaking, the height of the wave run-up in-



(a)



(b)

Fig. 3 (a) Schematic representation of breaking depth d_b , the distance from the slope to the interface at the point of gravitational instability. (b) Schematic representation of an inferred breaker depth is indicated based on the range obtained from video imagery.

creases with increasing wave amplitude. The data follow the trend given by the straight lines. For a steep slope, the internal wave run-up is approximately a linear function of wave amplitude, with a smaller run-up height for a milder slope. In Fig. 4 we compare the effect of slope inclination and maximum run-up height. The resultant data set shows an obviously complementary relationship to slope. The data for the supplementary slope angle show the same distribution. The results are in good agreement with those of Chen et al. [12], i.e., the resultant physical characteristics of an ISW on an impermeable planar slope in a stratified fluid can be illustrated by the mirroring criterion. Furthermore, when the impermeable slope is less, the height of maximum run-up is larger.

A specific linear addition can be found while considering a vertical slope ($\theta=90$ deg). The height of the wave run-up on a slope could be approximately double the incident amplitude of the ISW. On the other hand, superposition occurs when the ISW is transmitted toward a perpendicular cliff. This might be due to the effects of reduced gravity—the effective change in the acceleration of gravity acts on the fluid, which is in contact with a fluid of different density, as affected by buoyancy forces. In a reduced-gravity environment, surface tension leads the liquid to assume its solitary shape, meaning that capillary forces can cause the ISW to creep up the wall of an impermeable sloped plane.

As seen in Fig. 1, two types of ISW are generated. When the thickness of the fresh water in the small compartment is greater than that in main section of the flume, the interface moves clockwise when the gate was raised and vice versa. A depression-type internal wave evolves from the clockwise overturning of the interface; on the contrary, an elevation-type of internal wave is generated by counterclockwise interface overturning. We use the wave tank mechanisms pioneered by Kao et al. [4]. In our experiments, an elevation-type ISW propagates up normal slopes, while a depression-type ISW propagates on inverse slopes. There is apparent agreement in results showing run-up heights for $\theta=60$ deg (elevation-type waves) and $\theta=120$ deg (depression-type waves). The corresponding data distributions of ISW run-up for $\theta=50$ deg (elevation-type wave) and $\theta=130$ deg (depression-type wave) are similar, which verifies the “mirror criterion” phenomenon reported by Chen et al. [12].

The relationship between breaking depth and wave amplitude is shown in Fig. 5. The laboratory results indicated a strong linear trend such that the breaking depth increased with increasing wave amplitude. However, for wave amplitudes greater than 6cm, the plotted data are dispersed slightly around the regressed line. They show greater variations for large-amplitude waves. Interestingly, the breaking depth does not depend on the degree of uniform slope used in the test run.

An ISW with a small amplitude is reflected back away from an impermeable planar slope and keeps its amplitude and wave energy. The run-down velocities of large-amplitude waves are much stronger during the shoaling process. Owing to the intensive downslope flows, the Kelvin–Helmholtz instability incubated in the course of entraining the internal hydraulic jump, which then evolves into breaking due to rapid overturning. Energy loss induced by breaking might result in the dispersive data distribution. The energy loss of breaking and nonbreaking waves is responsible for the wave-slope interaction, but has no influence on the breaking depth, which is caused by the dynamism of an ISW. In other words, it is found that the trend for ISW-induced breaking depth on planar slopes is positively linear.

In previous reports [13–15], it is noted that linear energy loss and amplitude decay also occur due to bottom friction. Simple physical models describing the dynamics of the ISW become invalid at the breaking point, particularly those that assume linear behavior. Thus in the present experiments we ignored the physics of a breaking wave. That is, it is assumed that the breaking depth is caused by the kinetic energy of an incident ISW rather than changes resulting from the wave-slope interaction.

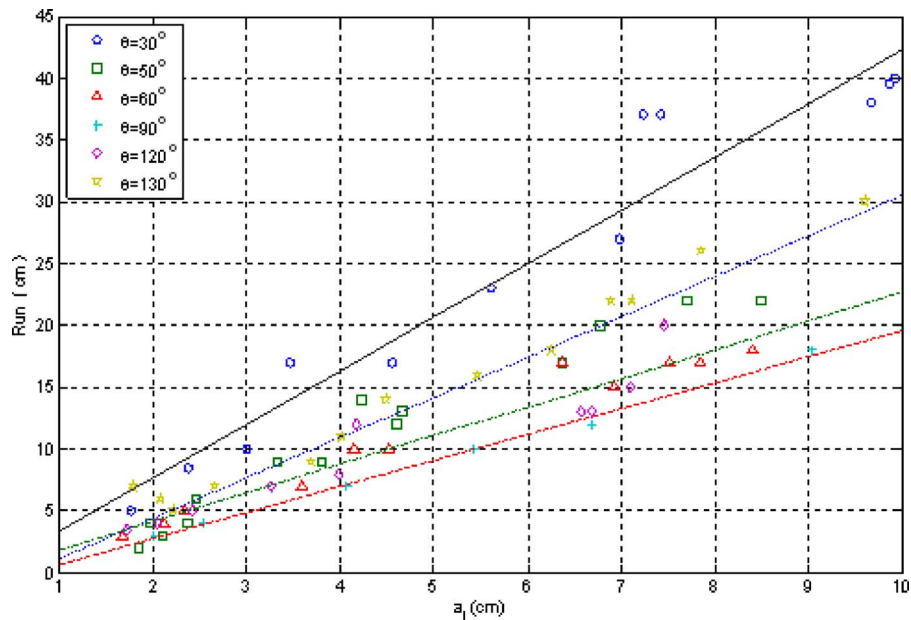


Fig. 4 Wave run-up height against the incident wave amplitude for various slopes, where (—) for $\theta=30$ deg; (· ·) for $\theta=50$ deg and $\theta=130$ deg; (---) for $\theta=60$ deg and $\theta=120$ deg; and (---) for $\theta=90$ deg

5 Conclusion and Discussion

The goal of this study is to explore experimentally the phenomenon of ISW run-up on smooth impermeable planar slopes based on a parameter representing the wave amplitude. The linear formulas derived therefrom should be as good as our existing understanding for estimating the run-up of nonlinear waves that break on slopes and better at estimating nonbreaking wave run-up. Maximum run-up of breaking and nonbreaking solitary waves on smooth impermeable planar slopes can be adequately predicted using the linear formula obtained from the present experimental work.

The results of laboratory experiments have been presented to illustrate and quantify the disturbance areas generated by the non-

linear internal wave forced interaction of a stratified two-layer fluid and a model continental shelf. It should be noted that in the present study we examined a larger range (<0.25) of nonlinearity than in previously studied laboratory models. The goal of this study was to find out whether the wave amplitude, the slope of the seabed, and the wave characteristics were important factors in the run-up distance.

We investigate both depression ISWs and elevation ISWs in a horizontal one-dimensional domain, in our exploration of the run-up effect in a two-layer fluid system. It was found that an ISW would not change its soliton feature. Chen et al. [12] said that:

An ISW could retain its soliton feature on a flat bottom (e.g., $\theta=0^\circ$). However, when it encountered a vertical structure (e.g.,

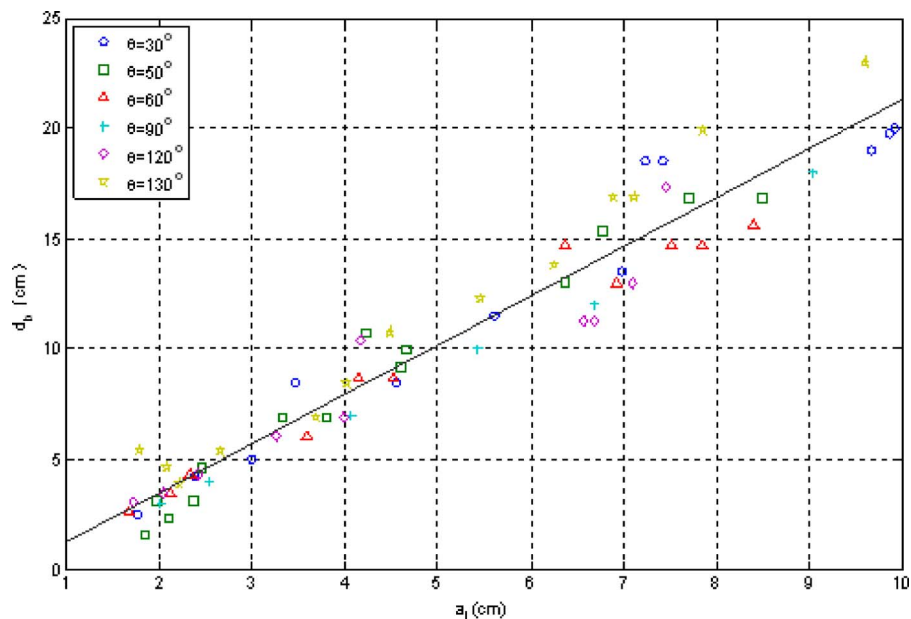


Fig. 5 Linear regression trends between breaking depth against incident wave amplitude

$\theta=90^\circ$), an incident ISW would not change its shape but it could be reflected as a nonbreaking soliton in a stratified two-layer fluid.

According to their mirror-image model, the uniform slope can be viewed as a flat mirror placed on the flat horizontal bottom. The ISW evolution on the slope is simply a reverse image opposite the virtual mirror. Furthermore, considering practical implication, we can speculate an ISW propagation toward a perpendicular cliff would result in the superpositioning on wave amplitude.

The movement of ocean waves in opposing directions might also occur at storm centers. The reflection of a swell at the shore is also ubiquitous. These wave-wave interactions are probably a result of cyclonic depressions on steep shelves along coastlines. More recently, it has been demonstrated that ocean microseisms are created in response to oceanic wave-wave interactions [16]. Interactions between internal waves and a variety of slopes, including steep normal slopes and inverse slopes, occur in many places such as in an estuary, a lake, or in the ocean. The amplitude of some internal waves can be large enough (up to 100 m or even more) to cause surface fluctuation. The experimental results presented in this paper are of benefit for future studies of internal wave hydrodynamics, and have practical implications in the field of earth science for investigating ISW microseisms during shoaling.

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