Slicing and dicing the 153-year record of monthly sea level at San Francisco, California using singular spectrum analysis

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O U T L I N E

1. A little history
2. The data
3. Preliminary calculations
4. The technique
5. Initial results
6. Effect of record length
7. Seasonal trends
8. The “hump” from ~1870 - 1900
9. Decrease in sea level since mid-1990’s
10. Conclusions

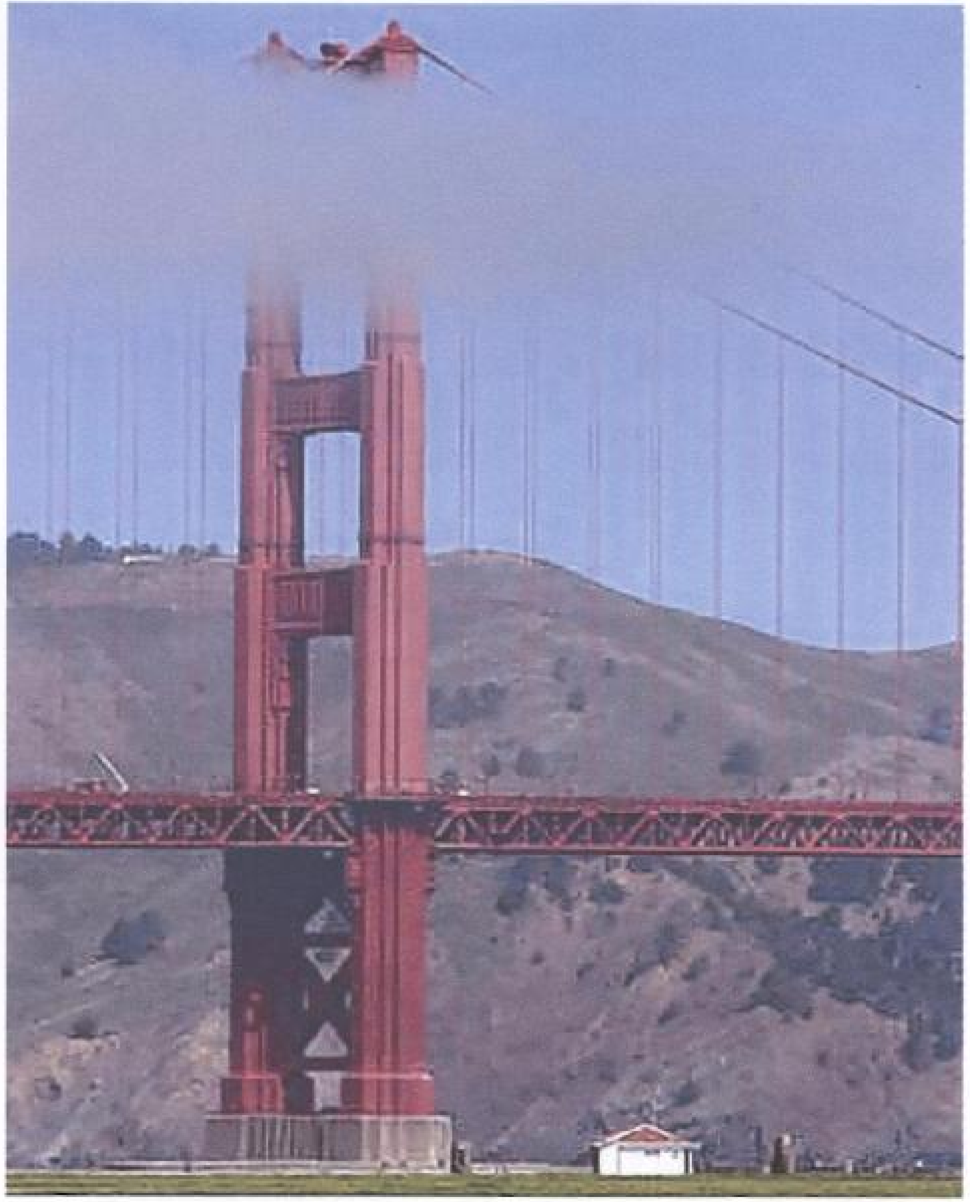


Fig. 1.-Golden Gate Area of California

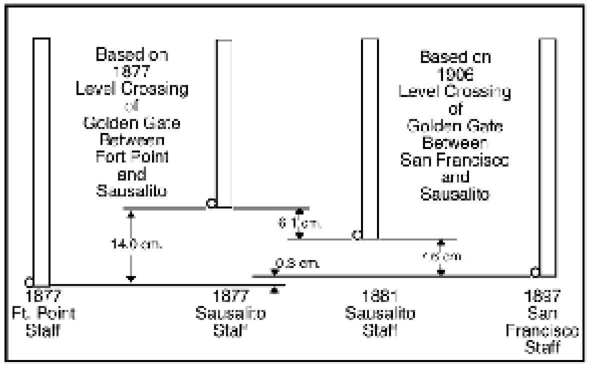


Fig. 2.-Leveling Between Tide Staffs at locations in Golden Gage

What is Singular Spectrum Analysis?

Singular spectrum analysis (SSA) is similar to the method of Principle Components in terms of its mathematical formulation. SSA is suitable for extracting information from short and noisy time series. The method unravels or extracts information embedded in delay-coordinate phase space by decomposing the sequence into elementary patterns of behavior in time by using so-called “data-adaptive filters”. These filters separate the time series into statistically independent components or modes, which can often be classified as trends, deterministic oscillations, or noise. The process of embedding the data in delay-coordinate phase space is equivalent to representing the behavior of the system by a succession of overlapping “views” of the time series through a sliding window whose length is called the window length or embedding dimension.

Singular Spectrum Analysis is based on a formal mathematical decomposition that consists of four steps:

- 1. The embedding step where a trajectory matrix is constructed from lagged versions of the original time series.
- 2. A Singular Value Decomposition (SVD) of the matrix formed by the product of the trajectory matrix and its transpose is performed, which corresponds to an eigenvalue problem that yields eigenvalues and eigenvectors.
- 3. The third step is grouping, which involves splitting the matrices from the SVD into groups and summing the matrices within each group
- 4. Transform the matrices into individual time series that can be summed to produce partial series, or if all of the individual series are summed, the original time series itself.

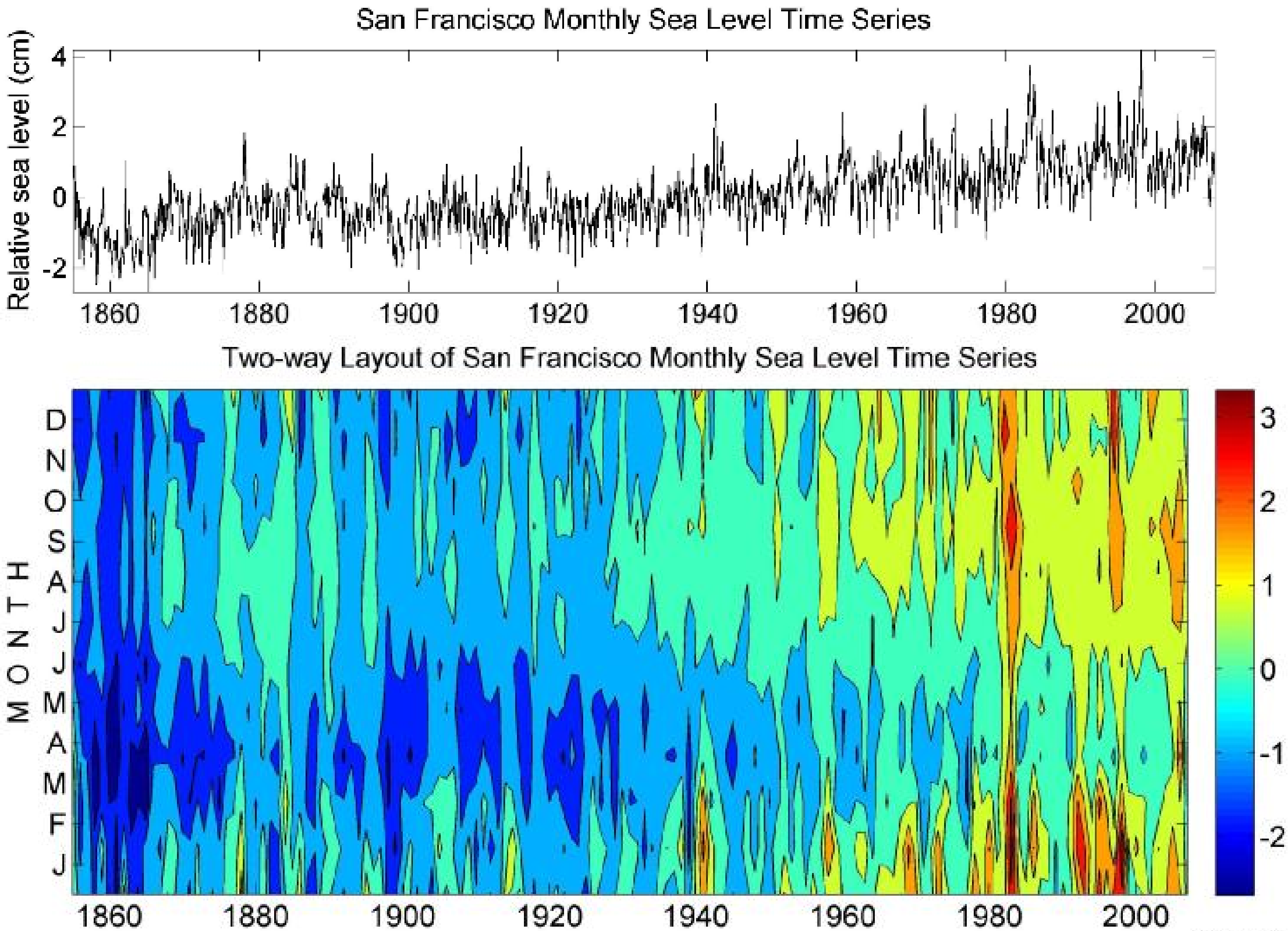
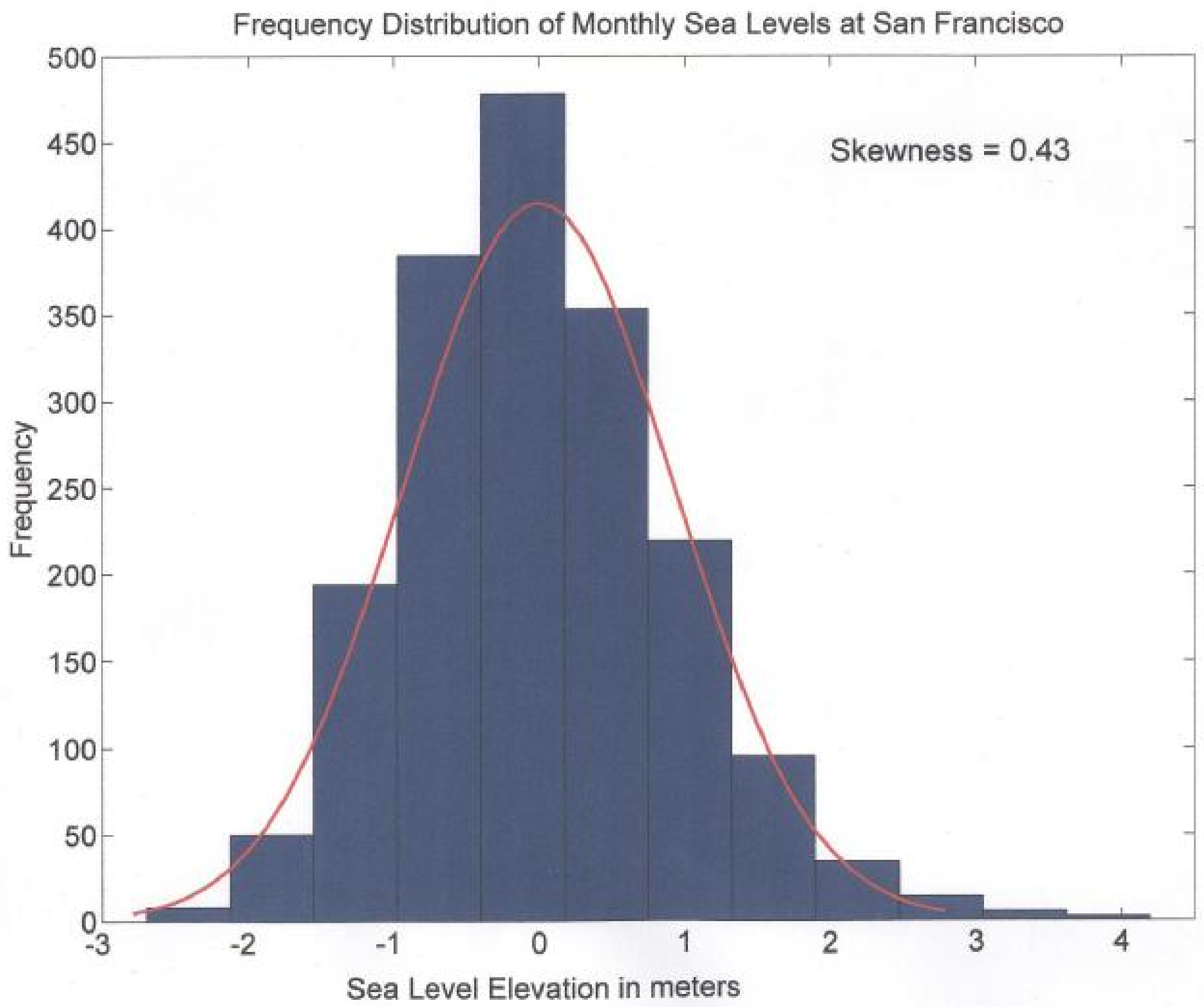
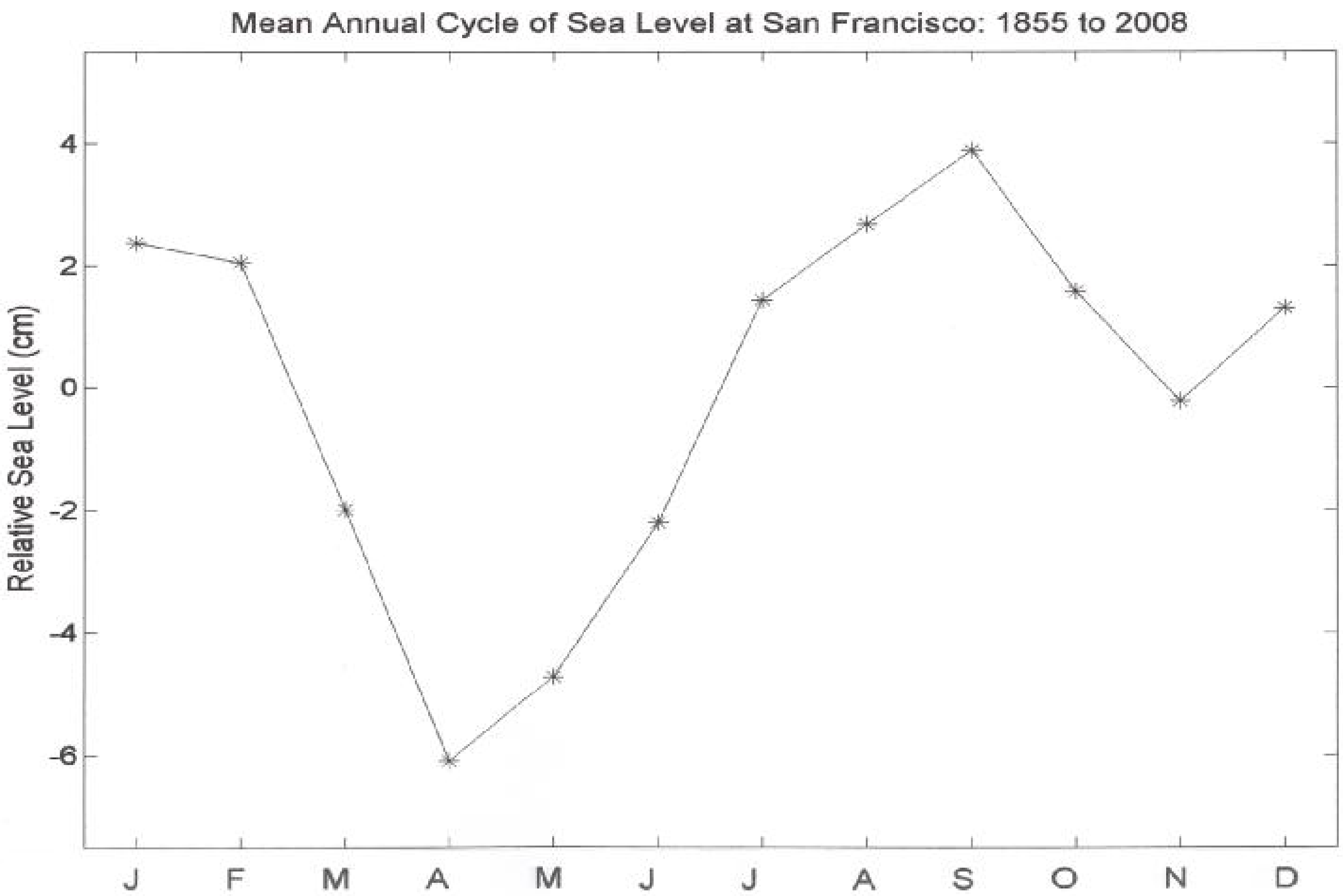

San Francisco Monthly Sea Level Time Series
Relative sea level (cm)
4
2
0
-2
1860
1880
1900
1920
1940
1960
1980
2000
Two-way Layout of San Francisco Monthly Sea Level Time Series
MONTH
D
N
O
S
A
J
J
M
A
M
F
J
3
2
1
0
-1
-2

Figure 1


Frequency Distribution of Monthly Sea Levels at San Francisco
Skewness = 0.43
Frequency
500
450
400
350
300
250
200
150
100
50
0
-3
-2
-1
0
1
2
3
4
Sea Level Elevation in meters



Mean Annual Cycle of Sea Level at San Francisco: 1855 to 2008
Relative Sea Level (cm)
4
2
0
-2
-4
-6
J
F
M
A
M
J
J
A
S
O
N
D

Figure 2

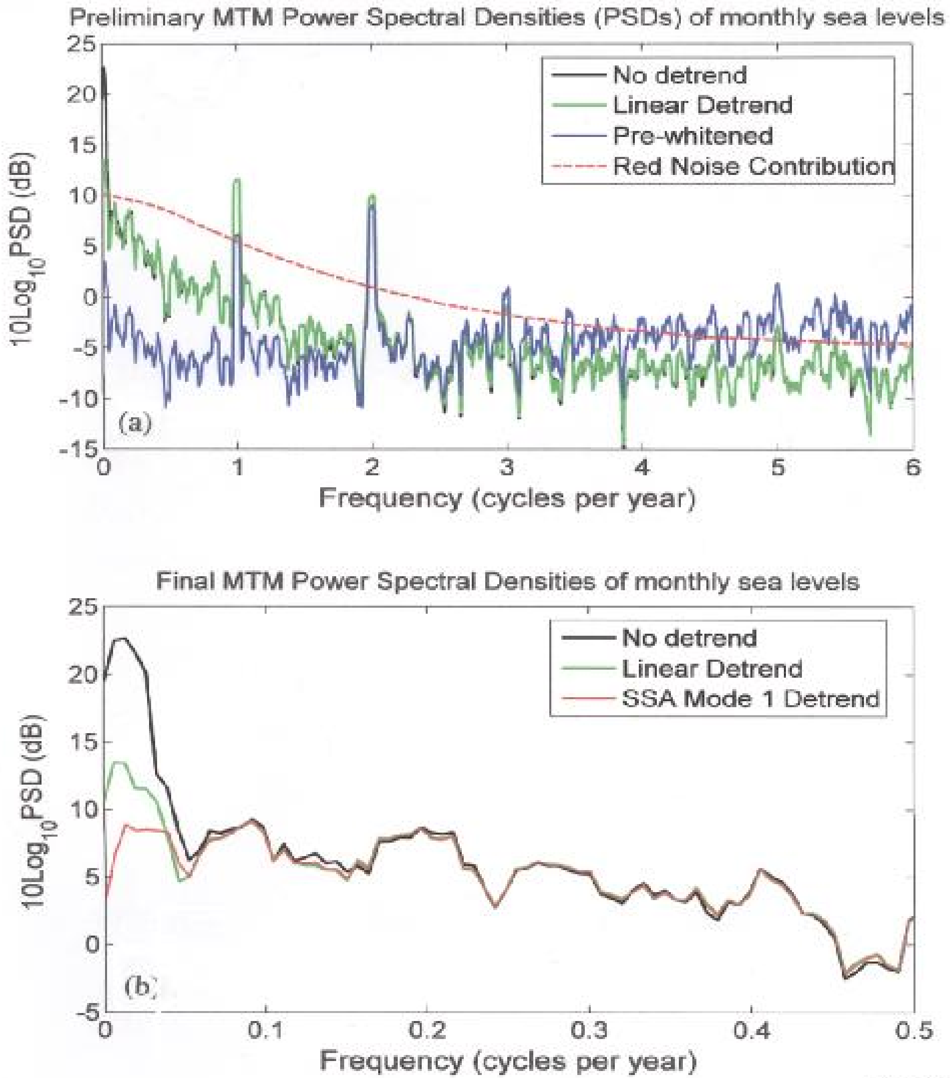

Preliminary MTM Power Spectral Densities (PSDs) of monthly sea levels
No detrend
Linear Detrend
Pre-whitened
Red Noise Contribution
10Log$_{10}$PSD (dB)
(a)
Frequency (cycles per year)
Final MTM Power Spectral Densities of monthly sea levels
No detrend
Linear Detrend
SSA Mode 1 Detrend
10Log$_{10}$PSD (dB)
(b)
Frequency (cycles per year)

Figure 3

TABLE 2. Summary of Ocean Signals in Tide Gauge Records and Comments on Ease of Removal From the Data

Signal	Amplitude, cm	Comments
Tides	100–200	easily removed by harmonic analysis or low pass filtering
Inverse barometer effect	1–10	mostly removed by adding 1.01 cm/mbar of atmospheric pressure (the exact validity of this correction remains questionable)
Geostrophic currents	1–100	result from many causes; some are easily removed (e.g., seasonal variability or El Niño) and others are difficult to remove (e.g., coastal upwelling or coastal trapped waves)
Coastal upwelling	10–20	difficult to remove since response to wind varies with stratification in the water column; significantly reduced by low-pass filtering
Coastal trapped waves	10–20	difficult to remove since amplitude and phase speed depend on quantities which cannot be adequately measured along propagation path; significantly reduced by low-pass filtering
Seasonal variability	1–40	easily removed from monthly average data by long-term average method or harmonic analysis
Low-frequency atmospheric forcing	1–4	easily removed by multivariate regression analysis
El Niño effect	10–50	easily removed by empirical orthogonal function analysis
Secular variability	1–10	easily removed by regression on linear trend but difficult to separate oceanic contributions from that due to crustal motion

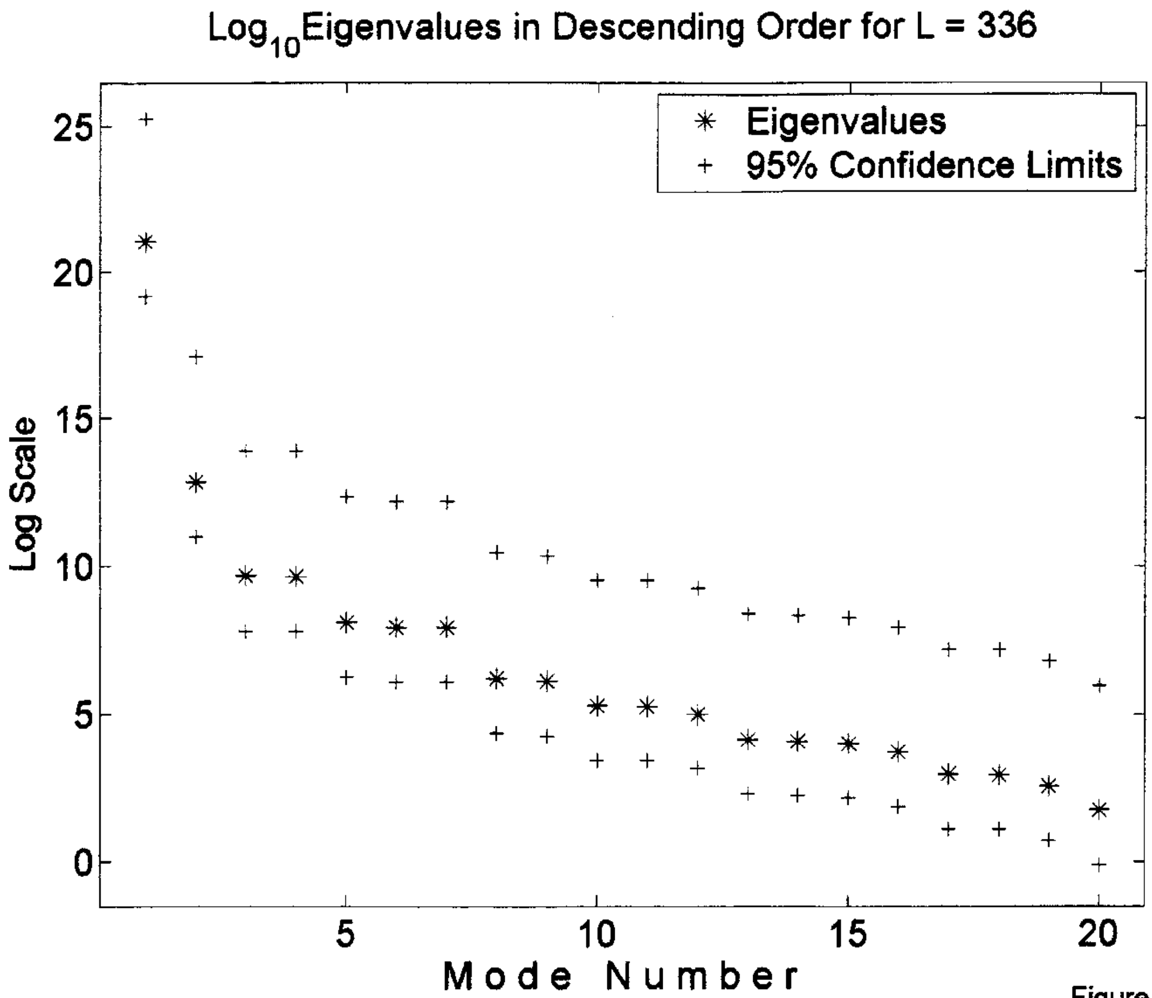

Log$_{10}$Eigenvalues in Descending Order for L = 336
Eigenvalues
95% Confidence Limits
Log Scale
0
5
10
15
20
25
5
10
15
20
Mode Number

Figure 4

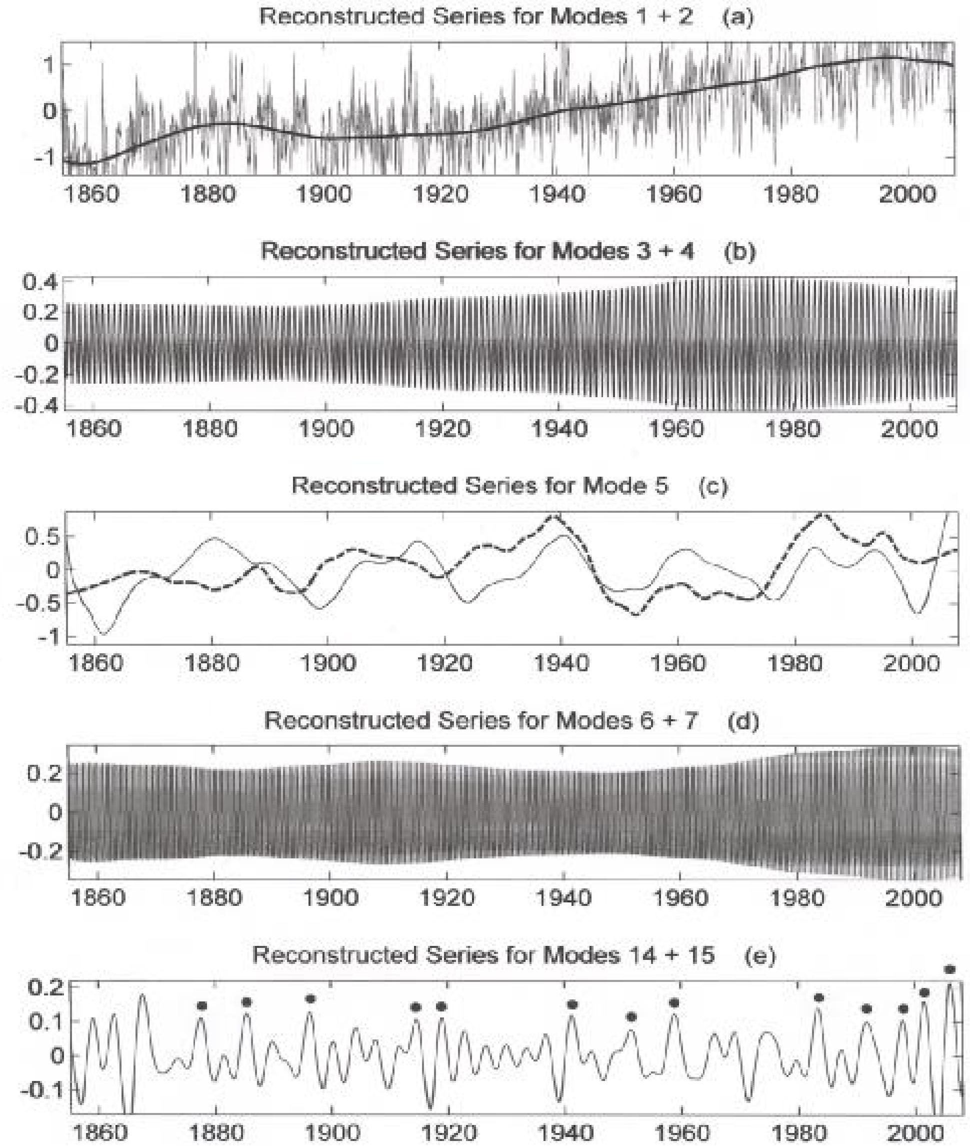
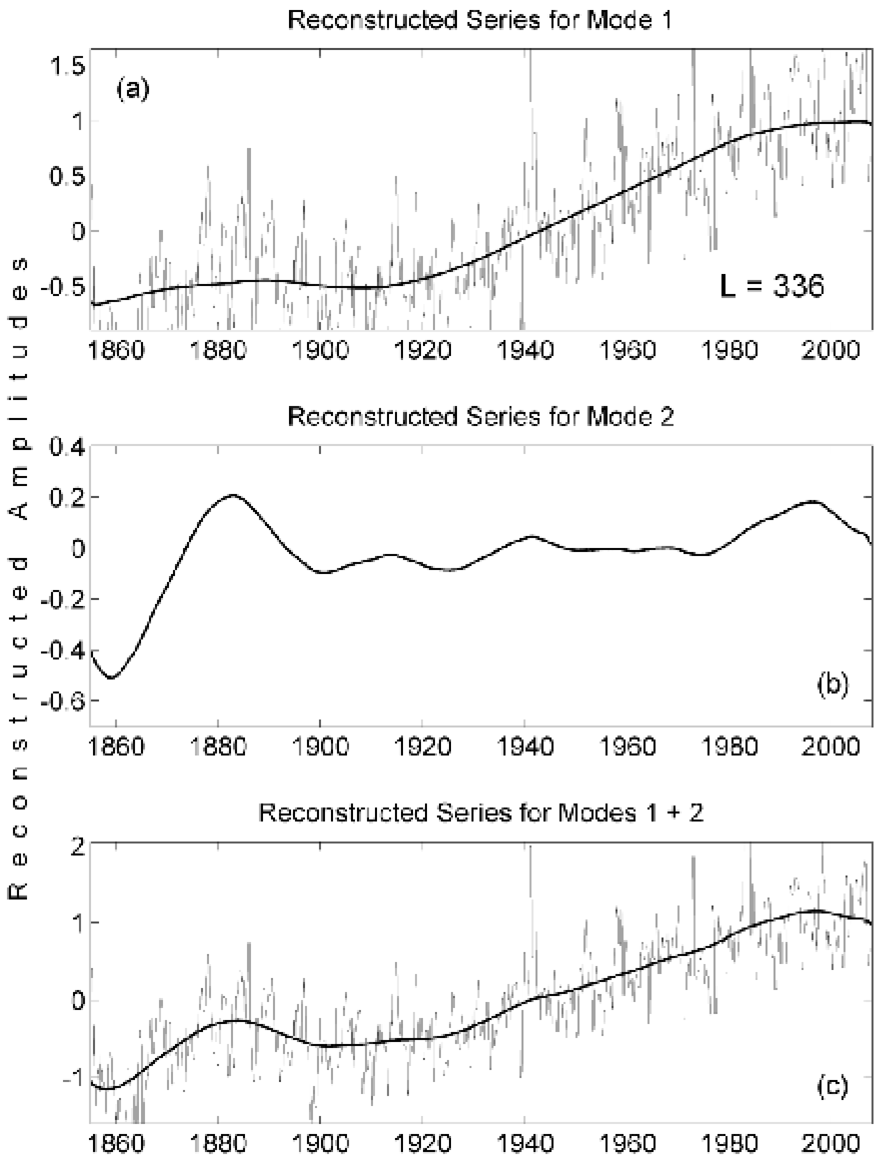
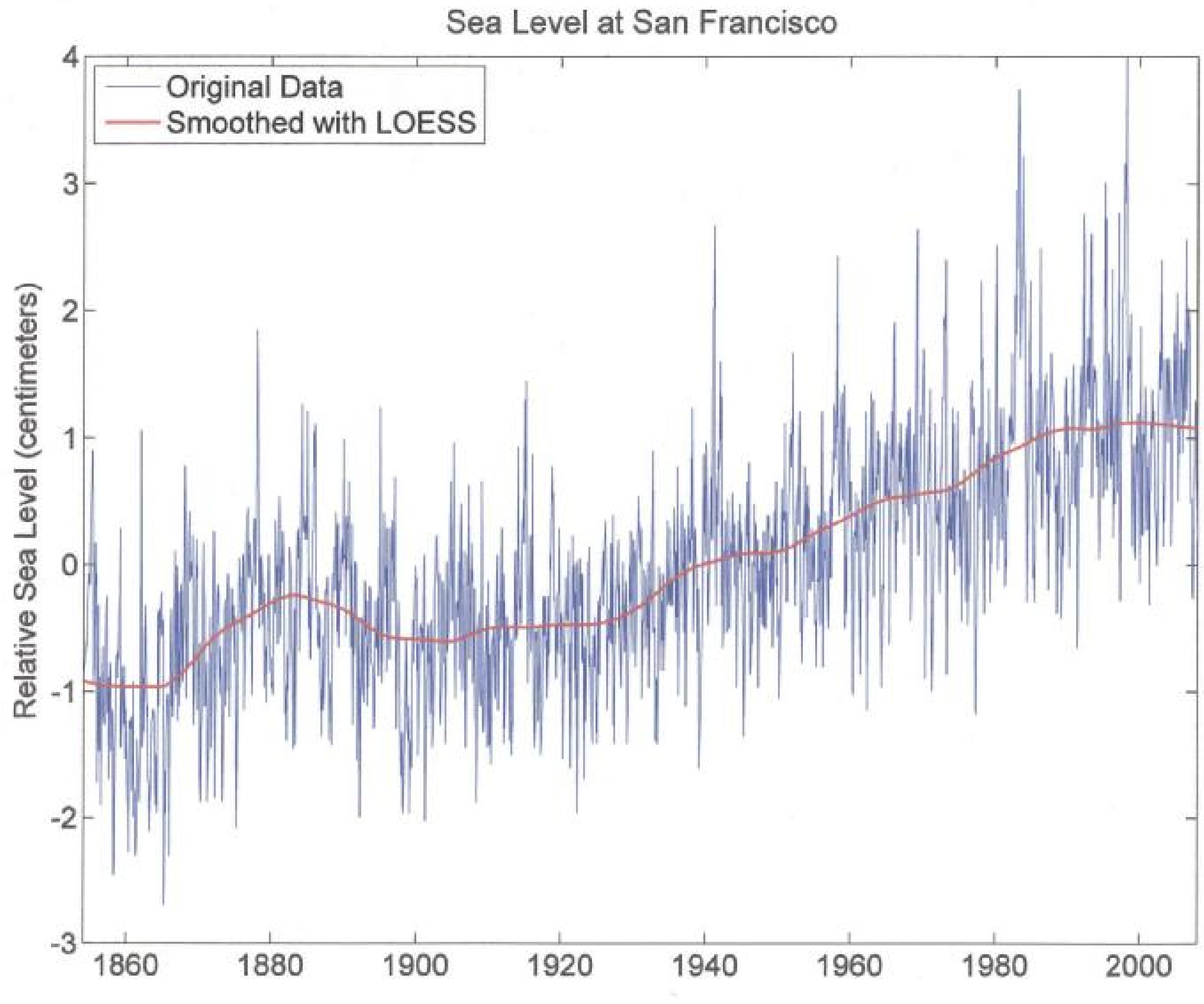

Reconstructed Series for Modes 1 + 2 (a)
Reconstructed Series for Modes 3 + 4 (b)
Reconstructed Series for Mode 5 (c)
Reconstructed Series for Modes 6 + 7 (d)
Reconstructed Series for Modes 14 + 15 (e)
1860
1880
1900
1920
1940
1960
1980
2000

Figure 5

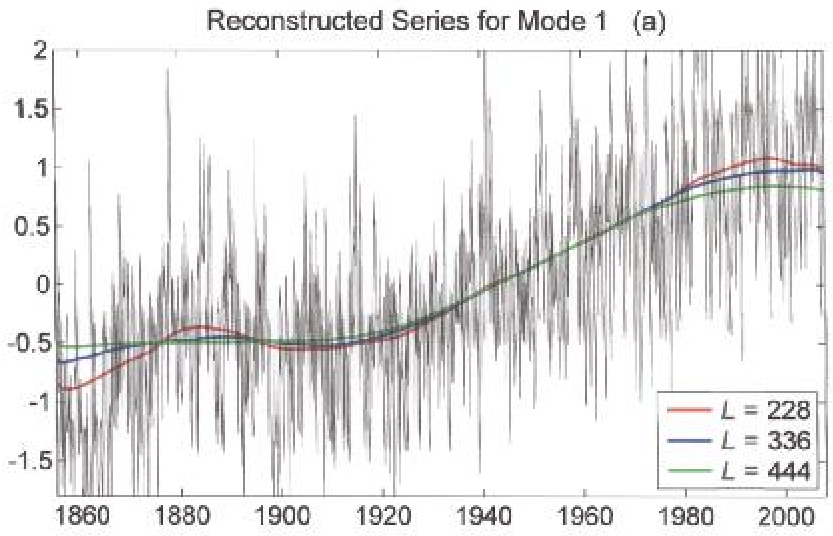


Reconstructed Series for Mode 1
(a)
L = 336
Reconstructed Series for Mode 2
(b)
Reconstructed Series for Modes 1 + 2
(c)
Reconstructed Amplitudes
1860
1880
1900
1920
1940
1960
1980
2000

Figure 6



Sea Level at San Francisco
Original Data
Smoothed with LOESS
Relative Sea Level (centimeters)
4
3
2
1
0
-1
-2
-3
1860
1880
1900
1920
1940
1960
1980
2000


Reconstructed Series for Mode 1 (a)
2
1.5
1
0.5
0
-0.5
-1
-1.5
1860
1880
1900
1920
1940
1960
1980
2000
L = 228
L = 336
L = 444

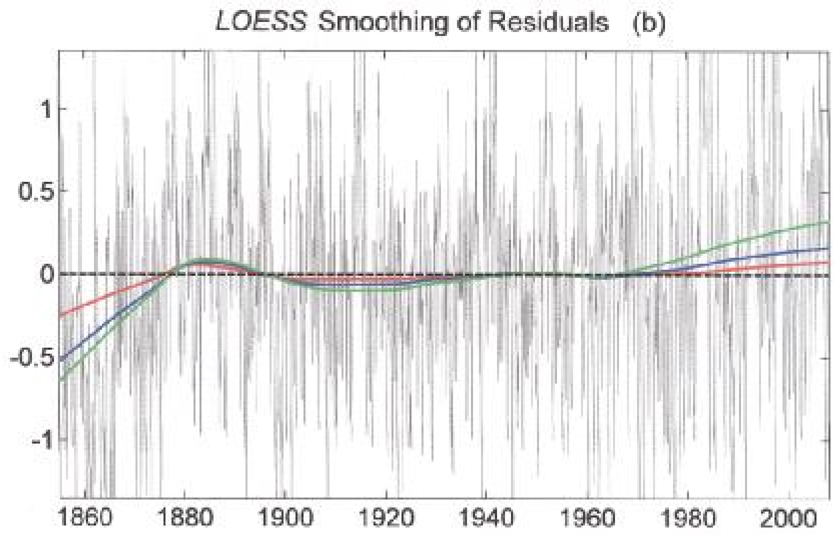
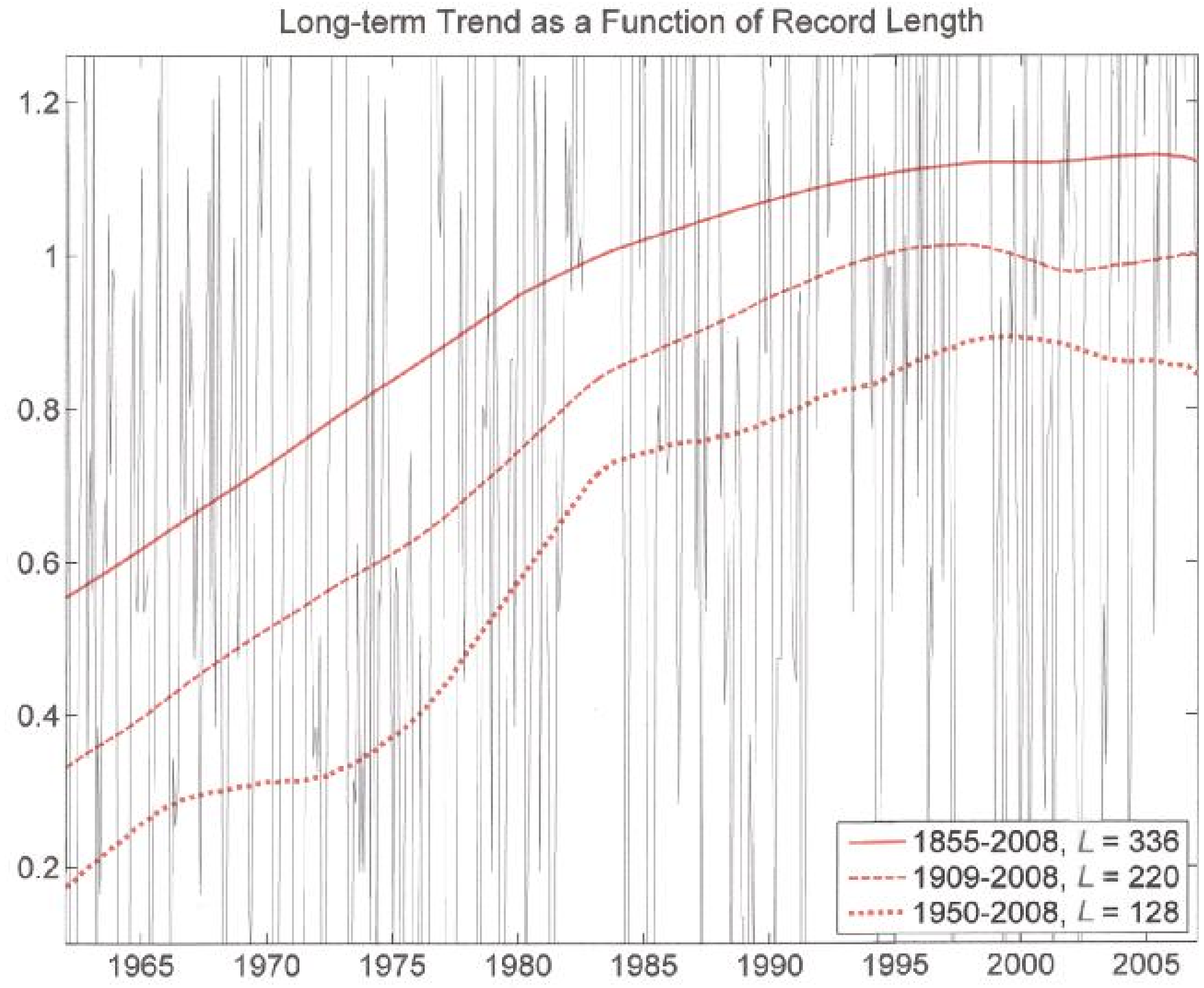
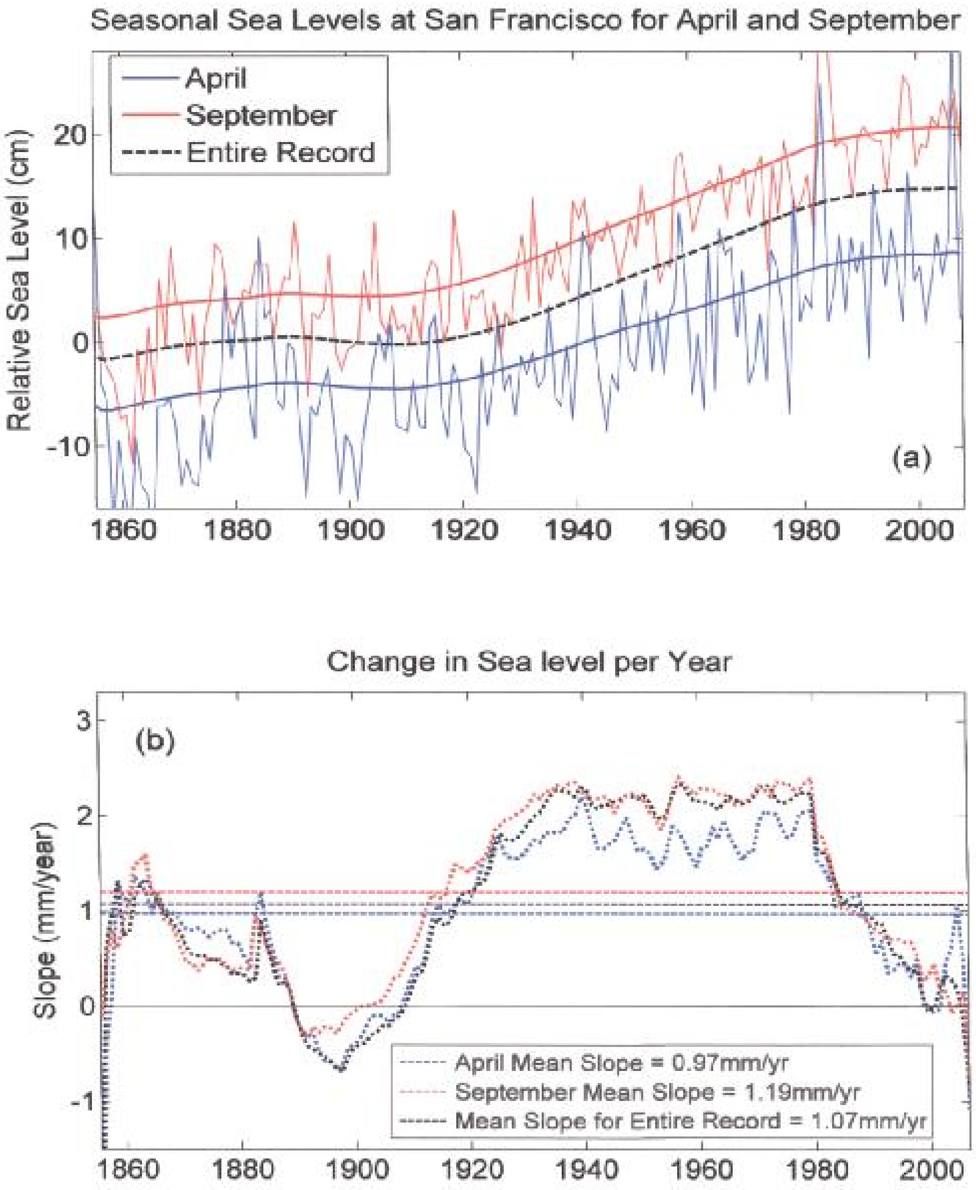

LOESS Smoothing of Residuals (b)
1
0.5
0
-0.5
-1
1860
1880
1900
1920
1940
1960
1980
2000

Figure 7



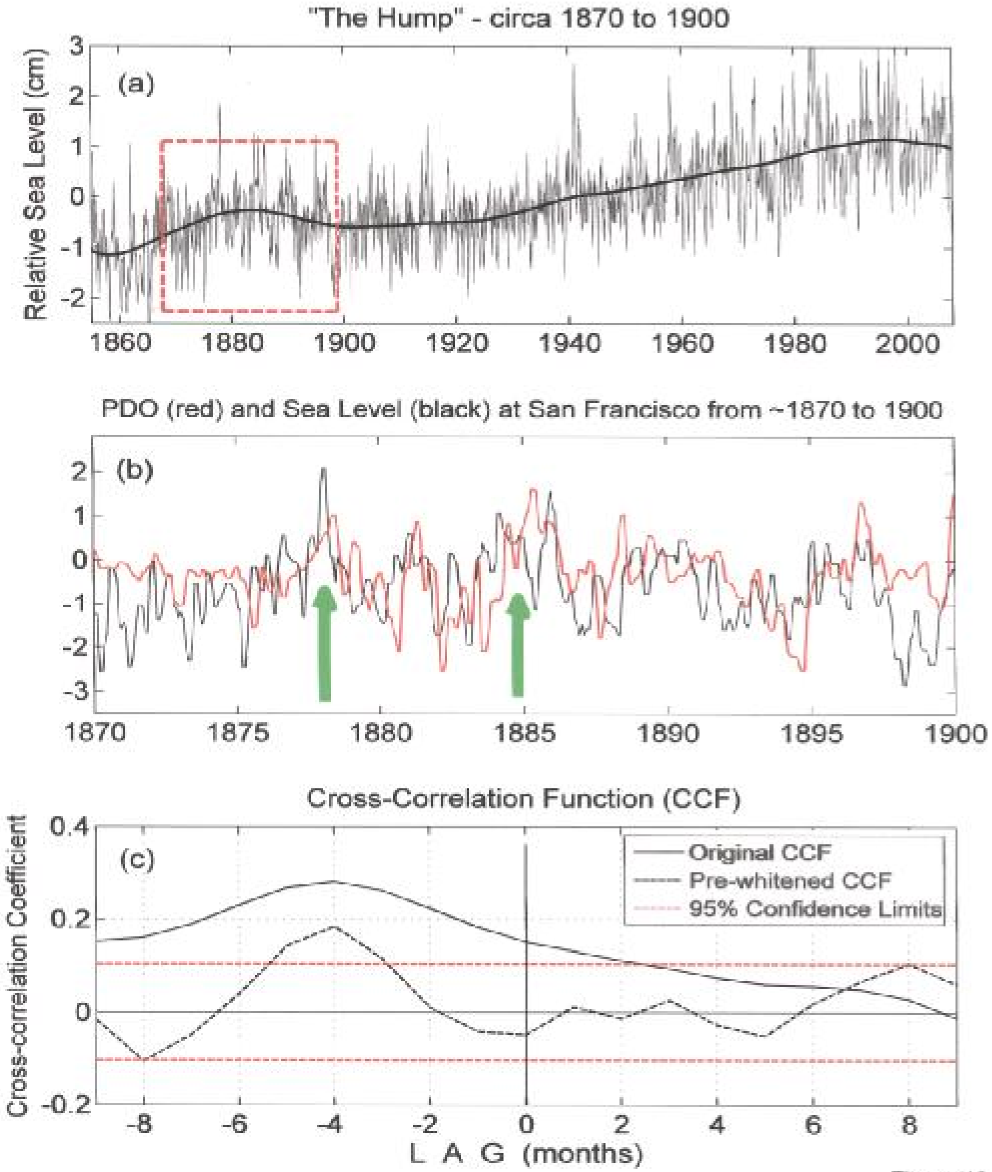
Long-term Trend as a Function of Record Length
1.2
1
0.8
0.6
0.4
0.2
1965
1970
1975
1980
1985
1990
1995
2000
2005
1855-2008, L = 336
1909-2008, L = 220
1950-2008, L = 128

Figure 8



Seasonal Sea Levels at San Francisco for April and September
April
September
Entire Record
Relative Sea Level (cm)
20
10
0
-10
(a)
1860
1880
1900
1920
1940
1960
1980
2000
Change in Sea level per Year
Slope (mm/year)
3
2
1
0
-1
(b)
April Mean Slope = 0.97mm/yr
September Mean Slope = 1.19mm/yr
Mean Slope for Entire Record = 1.07mm/yr
1860
1880
1900
1920
1940
1960
1980
2000

Figure 9



"The Hump" - circa 1870 to 1900
(a)
Relative Sea Level (cm)
3
2
1
0
-1
-2
1860
1880
1900
1920
1940
1960
1980
2000
PDO (red) and Sea Level (black) at San Francisco from ~1870 to 1900
(b)
2
1
0
-1
-2
-3
1870
1875
1880
1885
1890
1895
1900
Cross-Correlation Function (CCF)
(c)
Cross-correlation Coefficient
0.4
0.2
0
-0.2
Original CCF
Pre-whitened CCF
95% Confidence Limits
-8
-6
-4
-2
0
2
4
6
8
L A G (months)

Figure 10

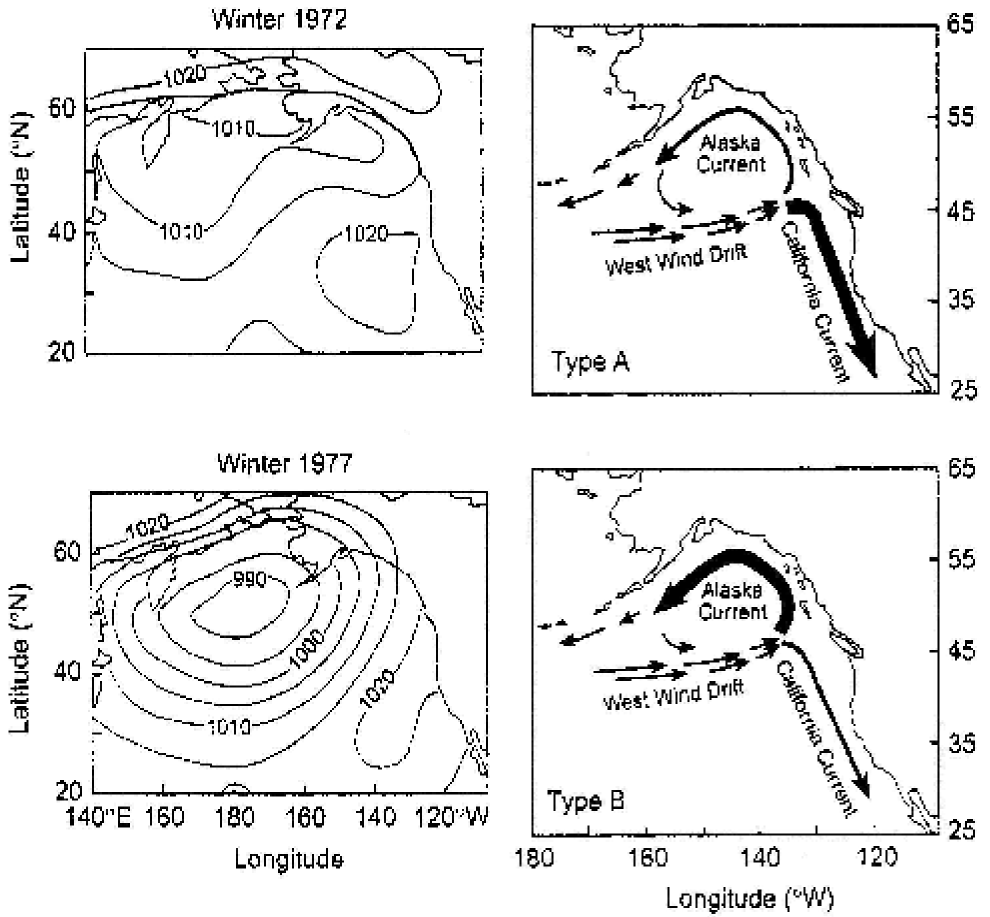
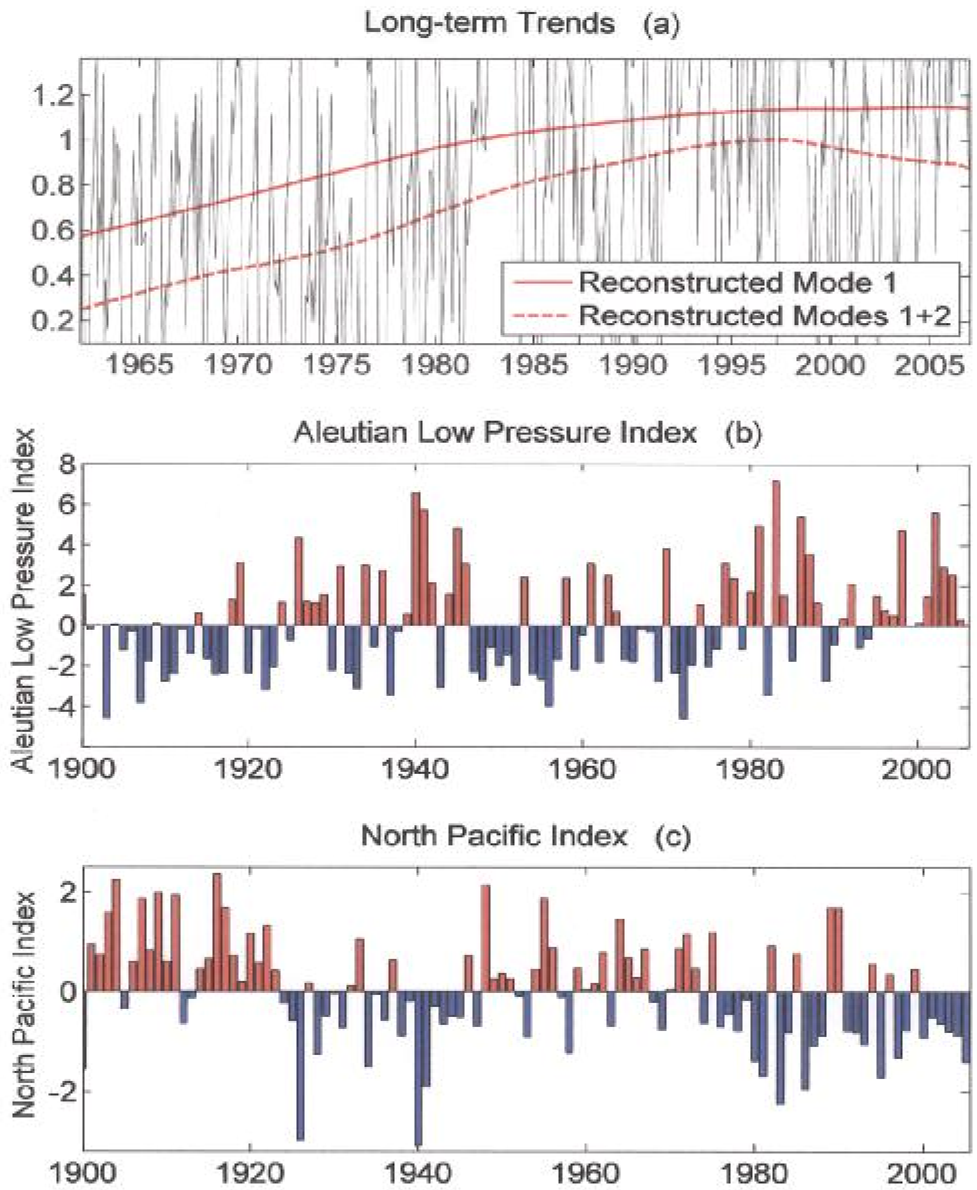
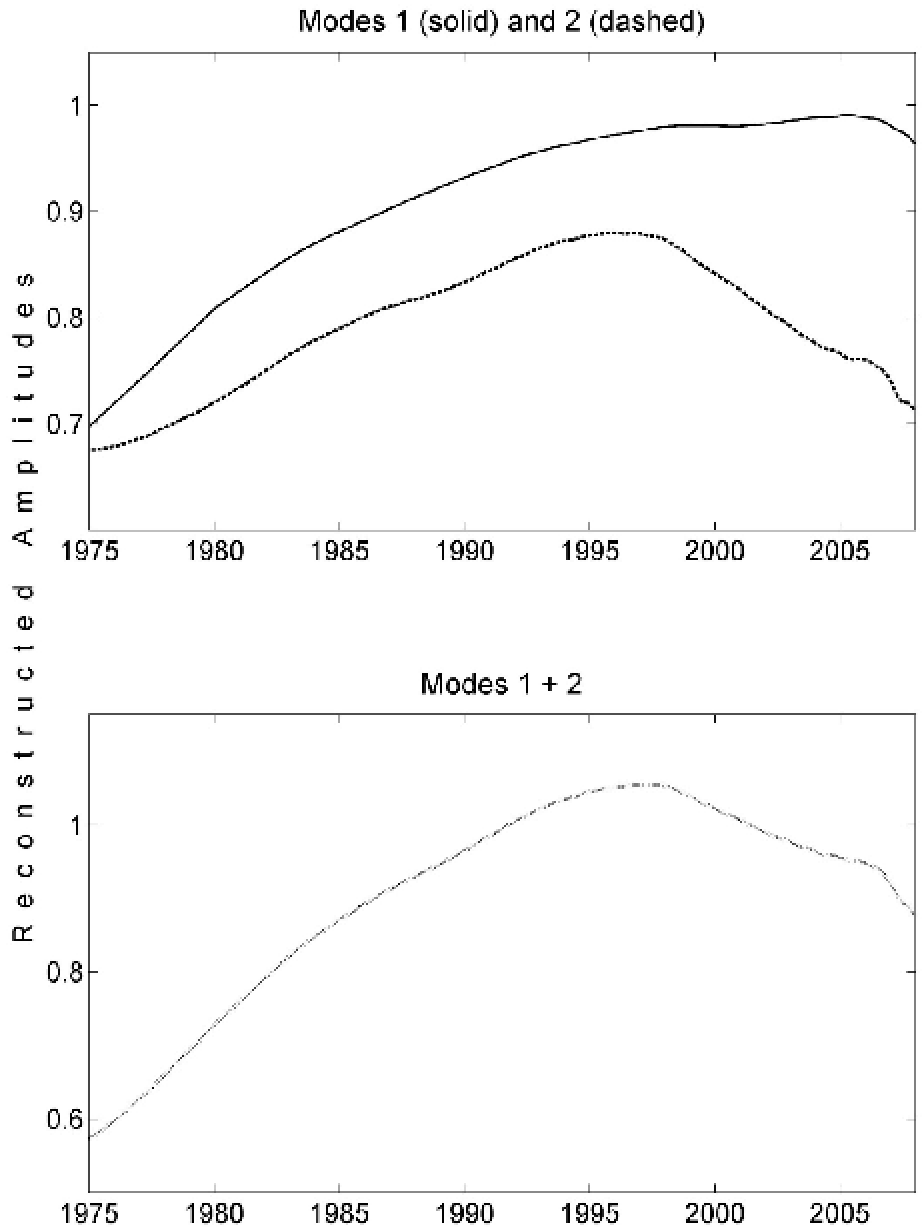


Fig. 9.08 Two alternating patterns of atmospheric circulation postulated by Hollowed and Wooster (1992). On the left are examples of weak and intense Aleutian low-pressure systems, and on the right the suggested changes in balance between the Alaska Current and the California Current. From Francis *et al.* (1998). Reproduced by permission of Blackwell Publishing Ltd.

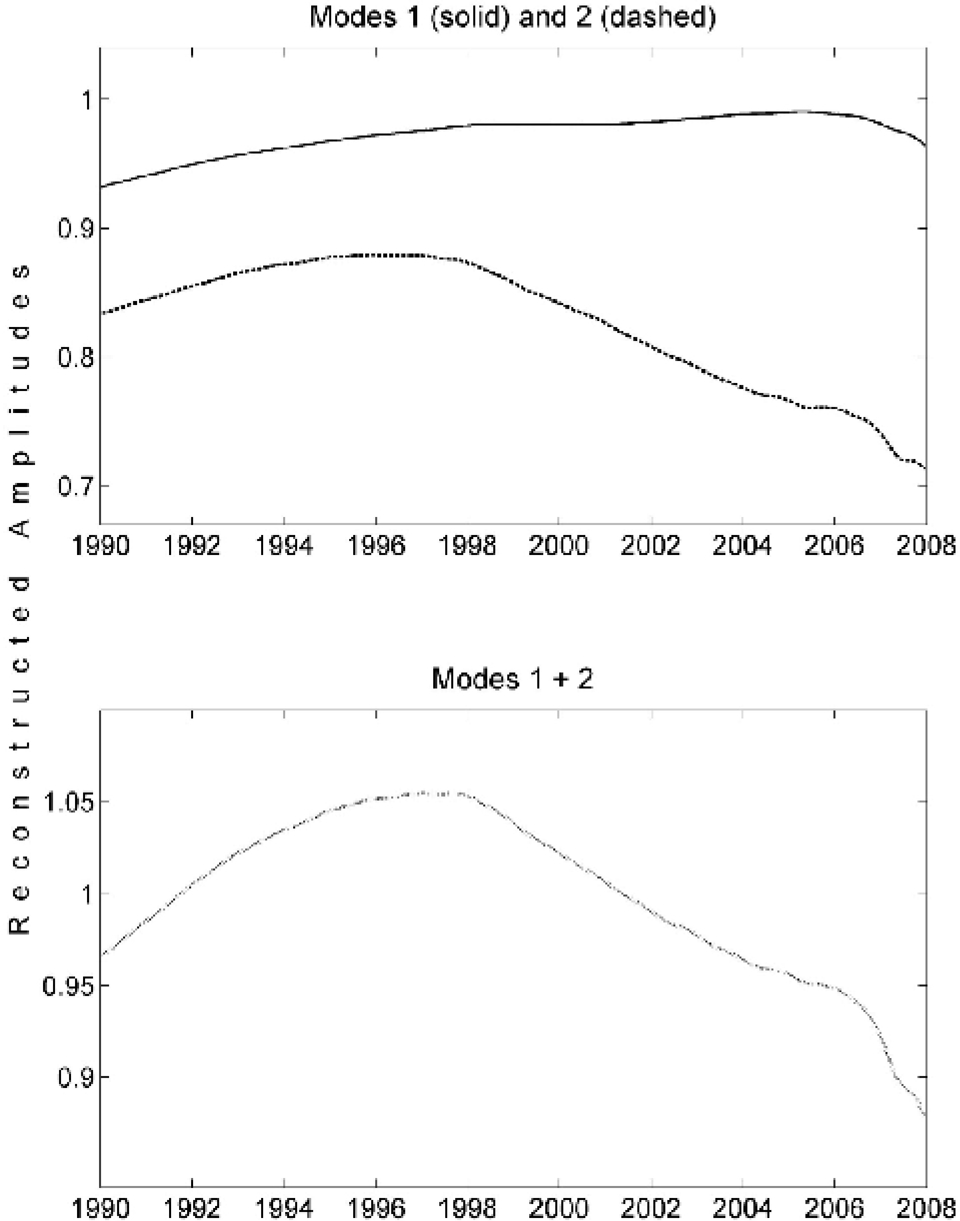


Long-term Trends (a)
1.2
1
0.8
0.6
0.4
0.2
1965 1970 1975 1980 1985 1990 1995 2000 2005
Reconstructed Mode 1
Reconstructed Modes 1+2
Aleutian Low Pressure Index (b)
Aleutian Low Pressure Index
8
6
4
2
0
-2
-4
1900 1920 1940 1960 1980 2000
North Pacific Index (c)
North Pacific Index
2
0
-2
1900 1920 1940 1960 1980 2000

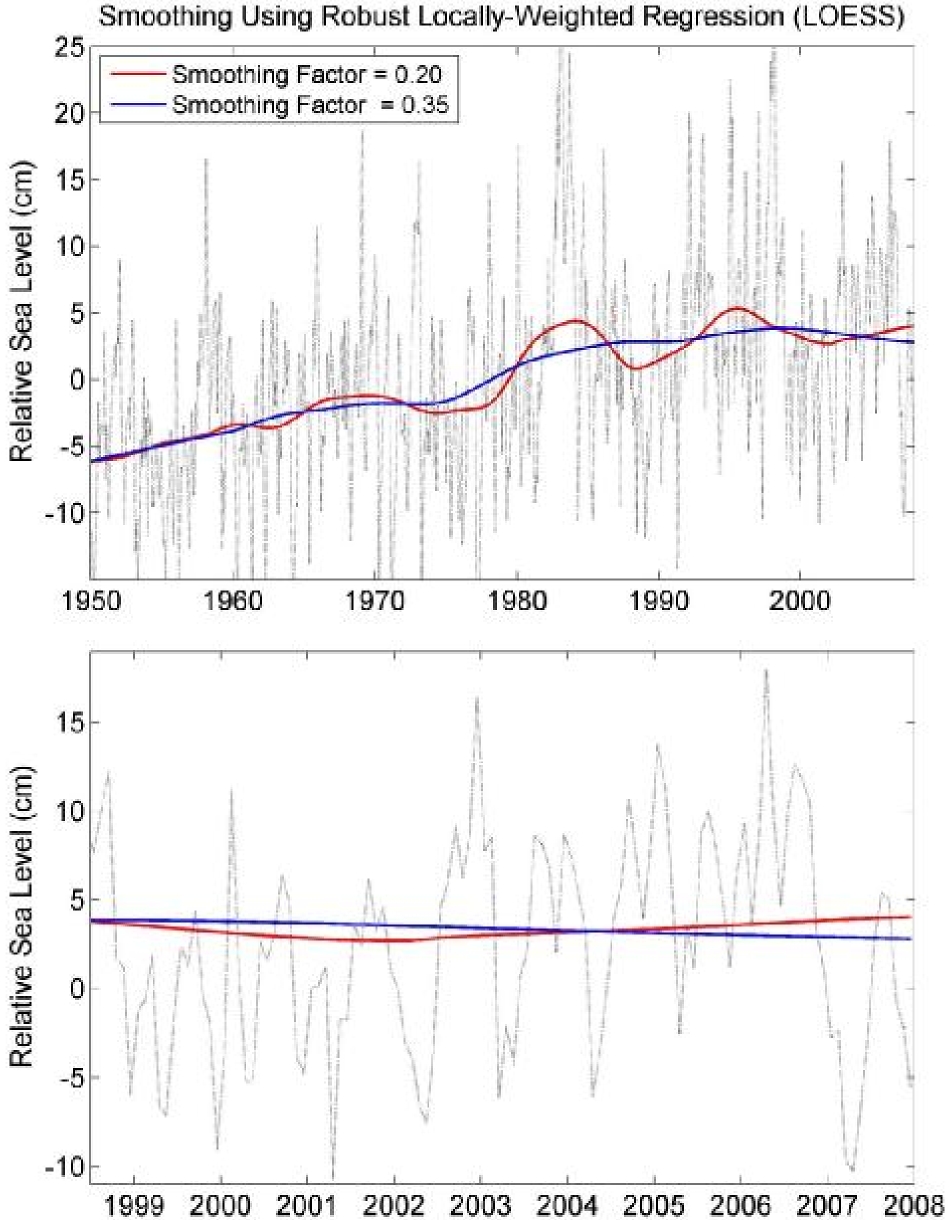
Figure 11


Modes 1 (solid) and 2 (dashed)
1
0.9
0.8
0.7
1975
1980
1985
1990
1995
2000
2005
Reconstructed Amplitudes
Modes 1 + 2
1
0.8
0.6
1975
1980
1985
1990
1995
2000
2005

Blowup (1975)


Modes 1 (solid) and 2 (dashed)
Reconstructed Amplitudes
1
0.9
0.8
0.7
1990 1992 1994 1996 1998 2000 2002 2004 2006 2008
Modes 1 + 2
1.05
1
0.95
0.9
1990 1992 1994 1996 1998 2000 2002 2004 2006 2008

Blowup (1990)


Smoothing Using Robust Locally-Weighted Regression (LOESS)
Smoothing Factor = 0.20
Smoothing Factor = 0.35
Relative Sea Level (cm)
25
20
15
10
5
0
-5
-10
1950
1960
1970
1980
1990
2000
Relative Sea Level (cm)
15
10
5
0
-5
-10
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008

Conclusions

- Tidal measurements have been made at San Francisco since 1854. It is the longest continuous record of sea level in existence.
- Singular Spectrum Analysis (SSA) reveals that the 1st mode corresponds to the long-term trend in rising level.
- Higher modes correspond to the annual cycle, the semi-annual cycle, the PDO, and ENSO events.
- The results indicate that SSA may provide an objective basis for extracting long-term trends.
- The structure of the long-term depends strongly on record length.
- Sea level rises faster in summer suggesting the importance of annual heating.
- Both the PDO and two major ENSO events contributed to the anomalous increase in sea level from circa 1870 to 1900.
- The long-term trend of increasing sea level has decreased or even reversed since the mid-1990s, and may be due to a sustained period of higher–than-average intensity in the Aleutian Low.